

Power Electronics Equivalent Sources, “Power Filtering” Analysis, Dc Conversion

P. T. Krein

Department of Electrical and Computer Engineering
University of Illinois at Urbana-Champaign

© 2011, 2015 Philip T. Krein. This work is licensed under a

Creative Commons Attribution-NonCommercial-ShareAlike 4.0 International License except certain photos as labeled.



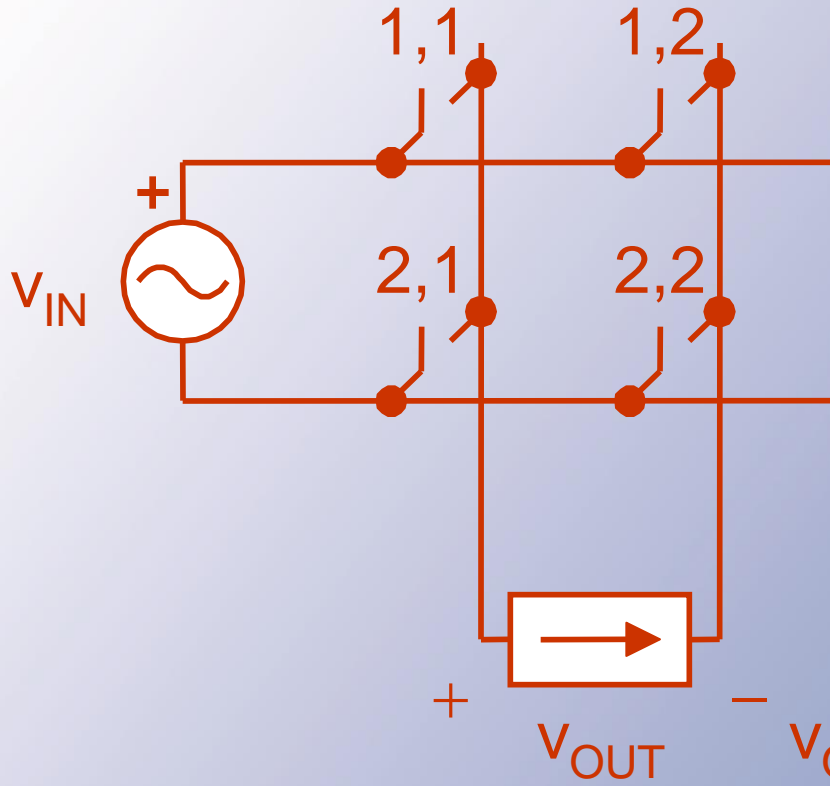
Equivalent Sources

When a switch matrix operates to satisfy KVL and KCL, many of the waveforms become well defined.

Example: Matrix 2x2 ac voltage to dc current converter.
The output must be $+V_{in}$, $-V_{in}$, or zero.



Equivalent Sources



$$V_{OUT} = \begin{cases} +v_{in} & \text{1,1 + 2,2 on} \\ -v_{in} & \text{2,1 + 1,2 on} \\ 0 & \begin{cases} \text{1,1 + 1,2 on} \\ \text{2,1 + 2,2 on} \end{cases} \end{cases}$$

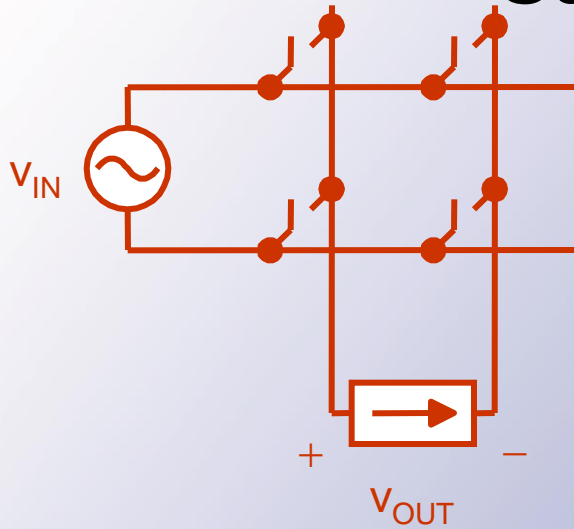
Equivalent Sources

- If switch action is specified, the output waveform becomes fully determined.
- We can treat the waveform as an ideal source (with an unusual shape).

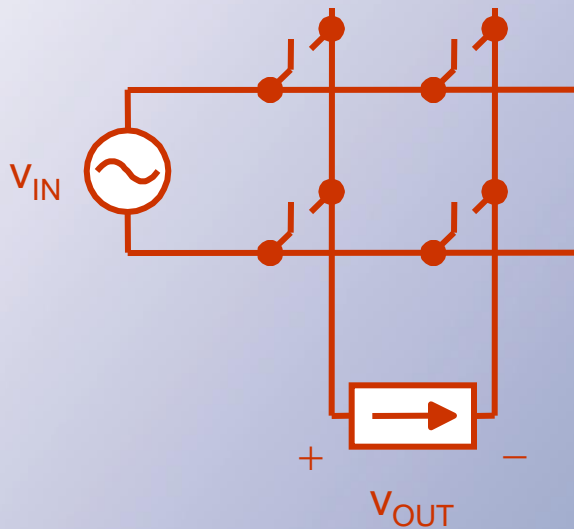
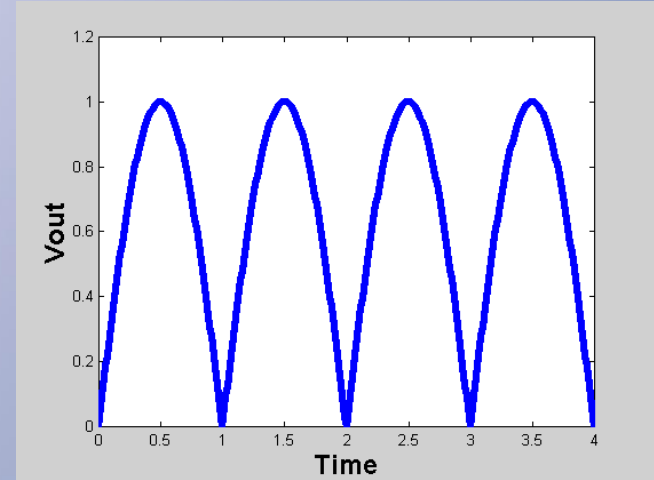
Sample Cases

- Full-wave rectifier
- Phase-delayed rectifier
- Inverter into an ac current source
- 60 Hz 3 ϕ to 60 Hz 1 ϕ conversion
- 60 Hz to 180 Hz

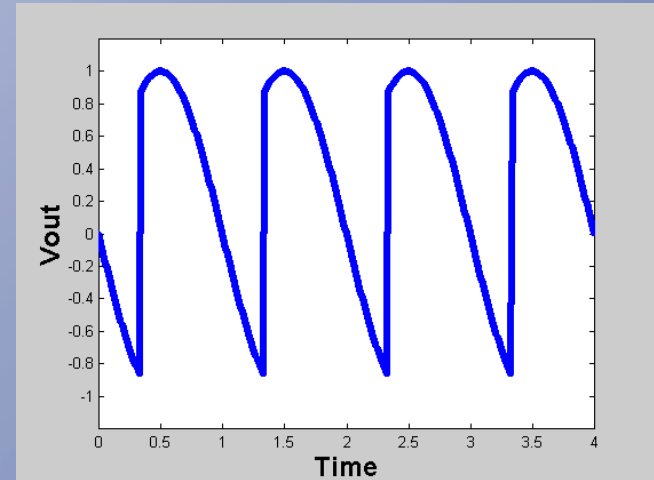
Sample Cases



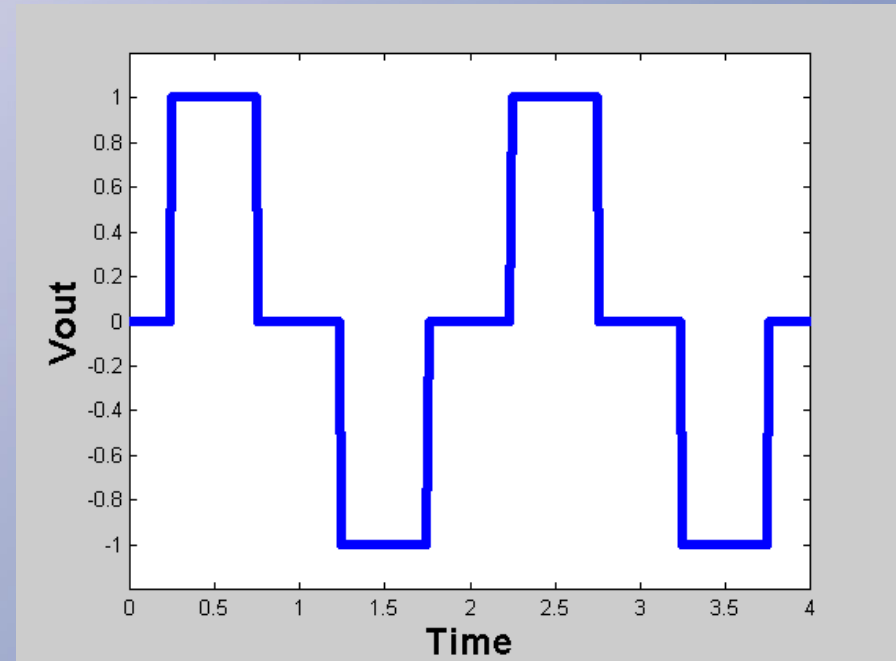
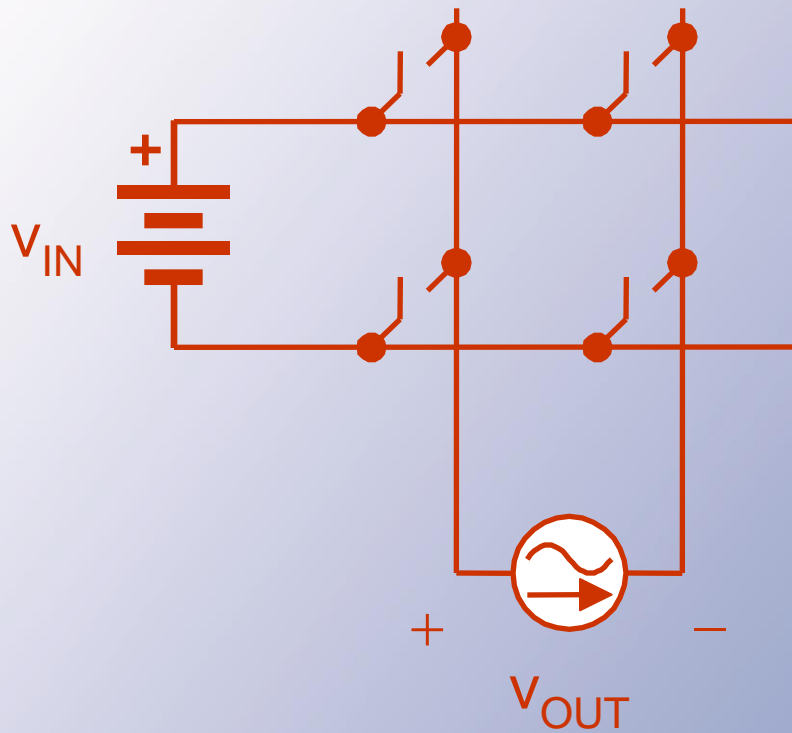
0° delay



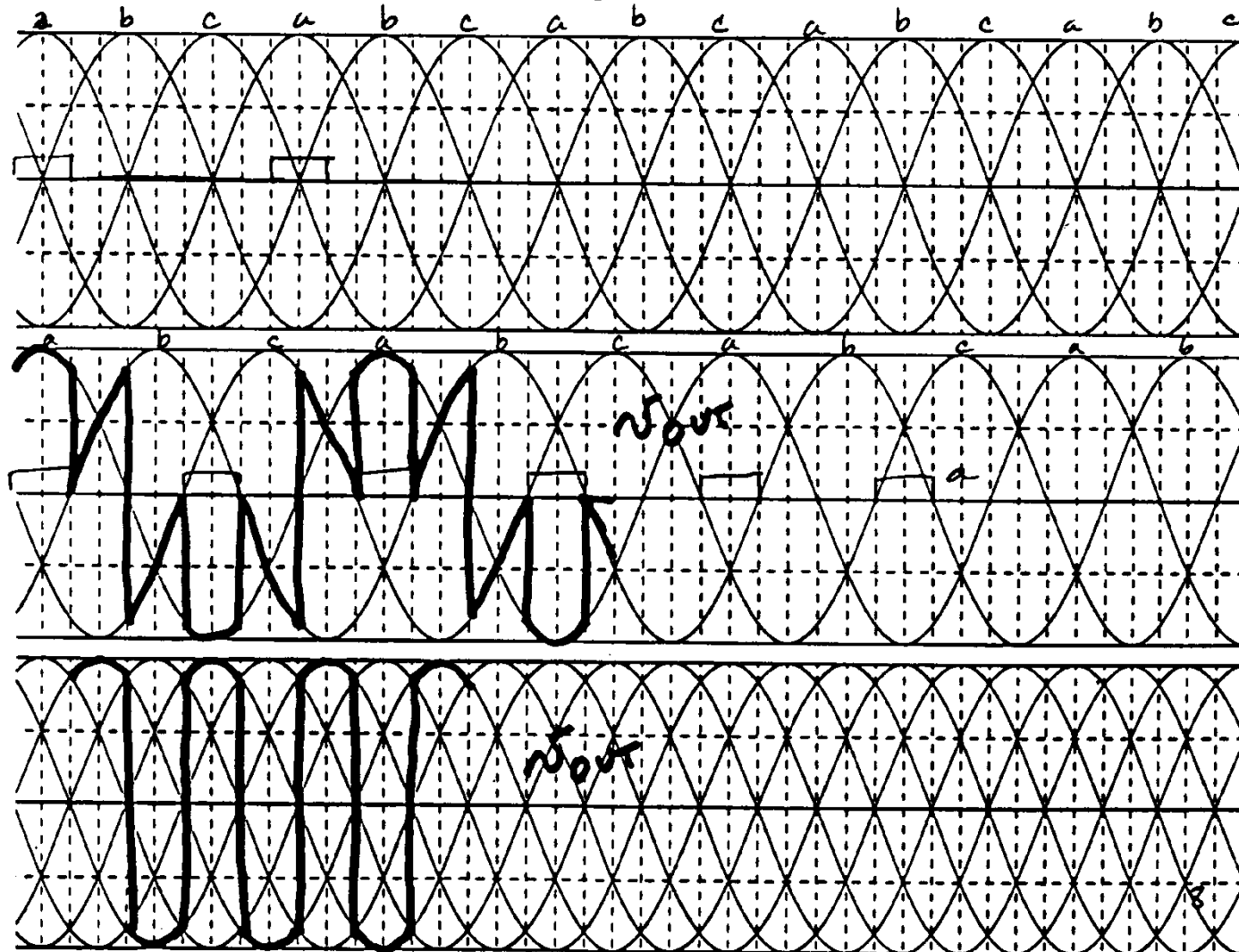
60° delay



Sample Cases

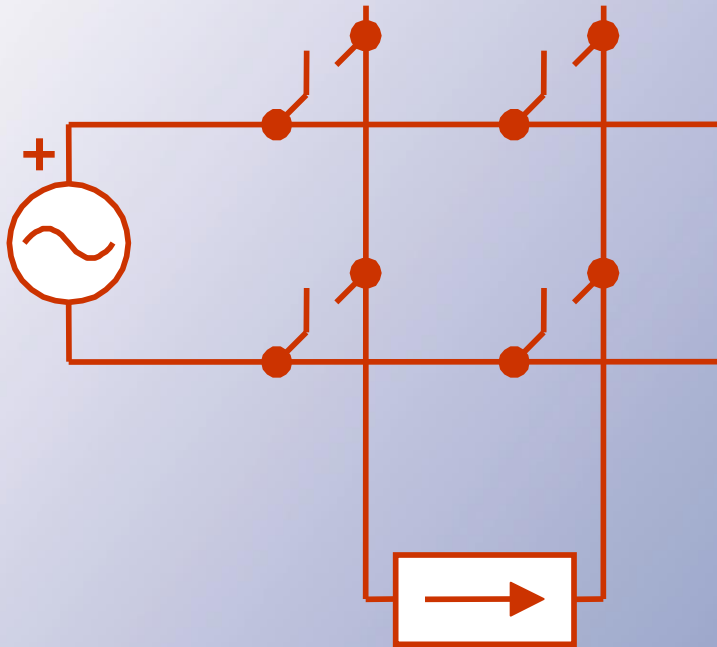


Sample Cases



Equivalent Sources

Any of those waveforms can be a source.



Equivalent Sources

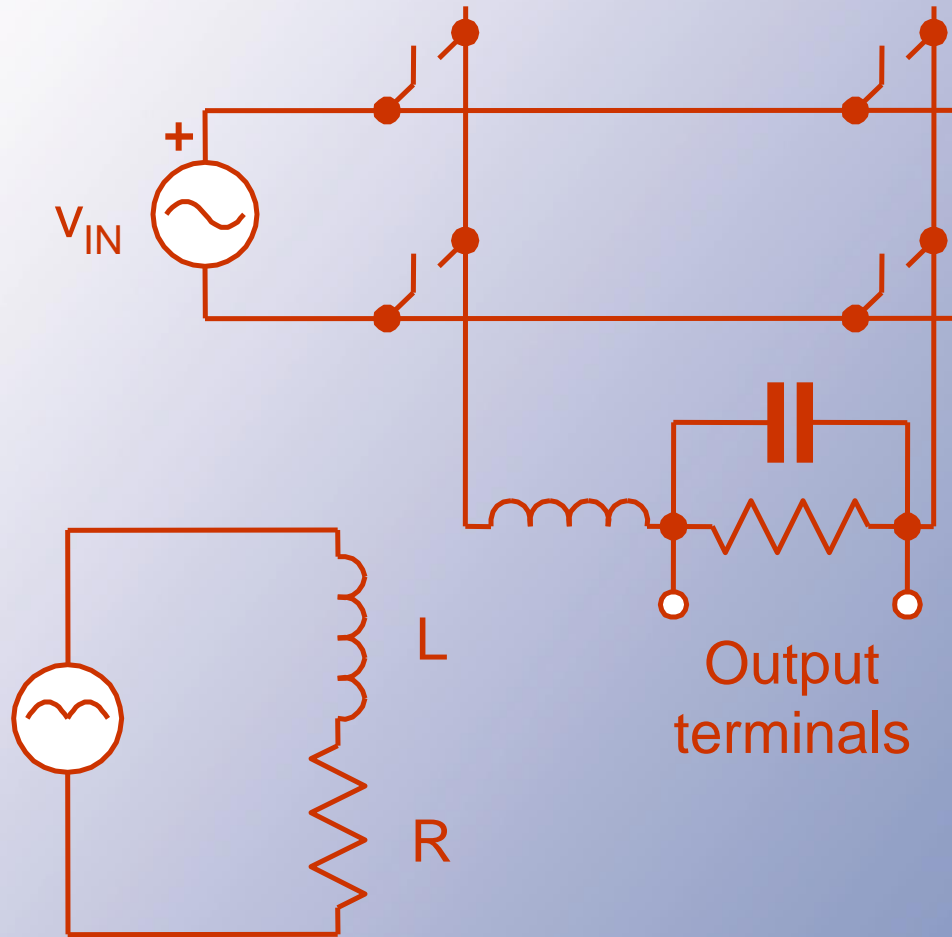
- Equivalent sources can be a powerful tool:
 - Many converters act like an equivalent source in a linear circuit
 - We can represent a source as a combination of Fourier components

Equivalent Sources

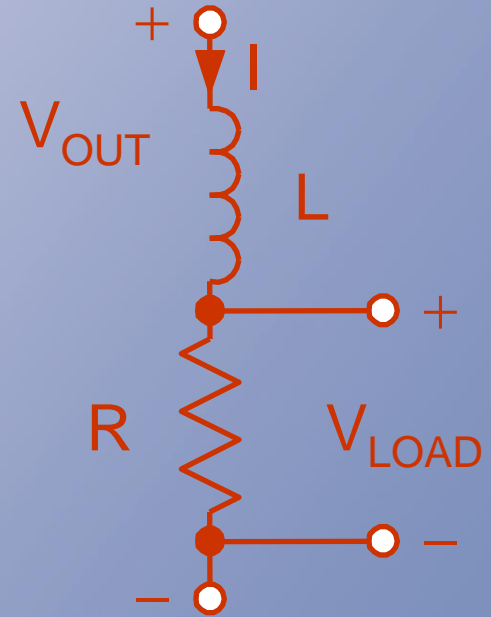
- With a source in a linear circuit, analysis, filter design, etc. can proceed along familiar lines.
- This is a common way to design interfaces for rectifiers and inverters.

Equivalent Sources

Example:

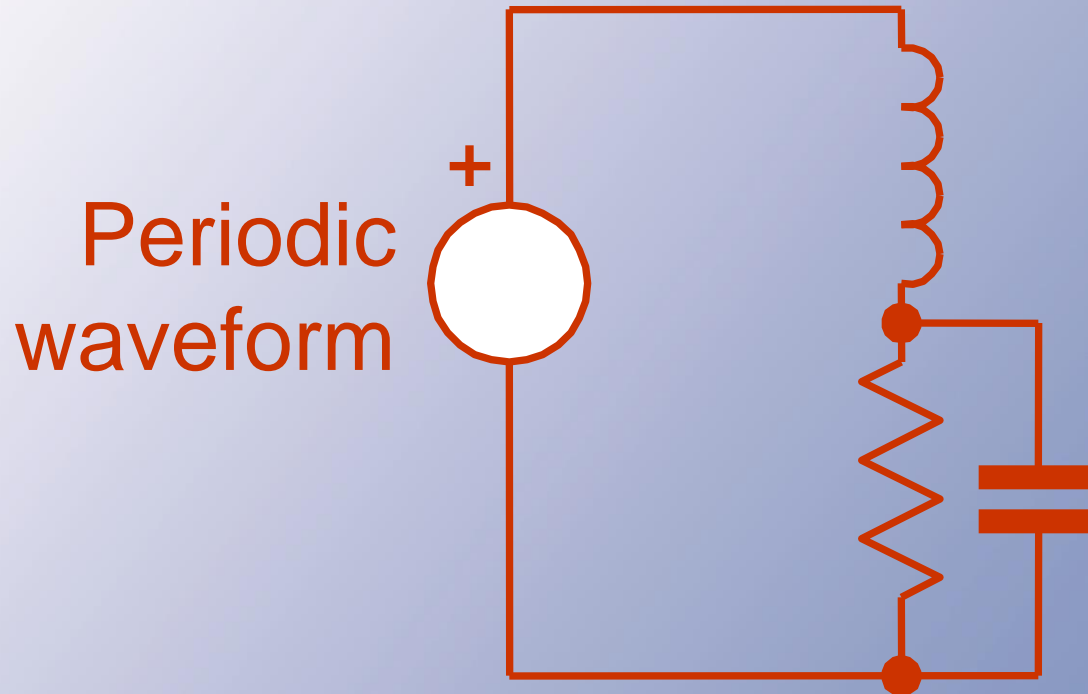


Ignore capacitor for a moment:



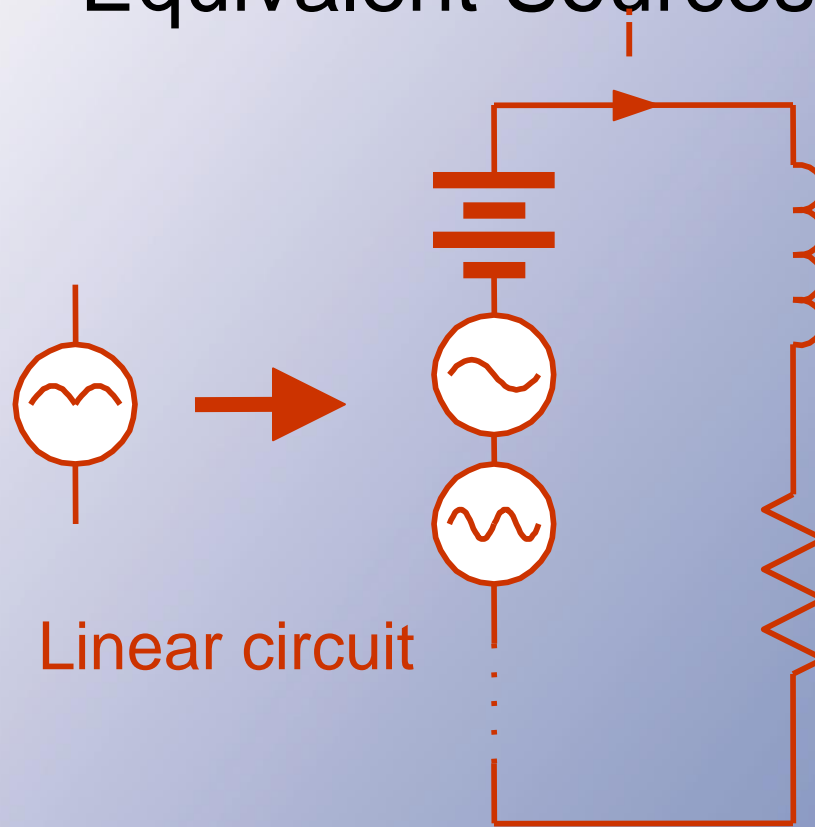
We know V_{OUT}

Equivalent Sources



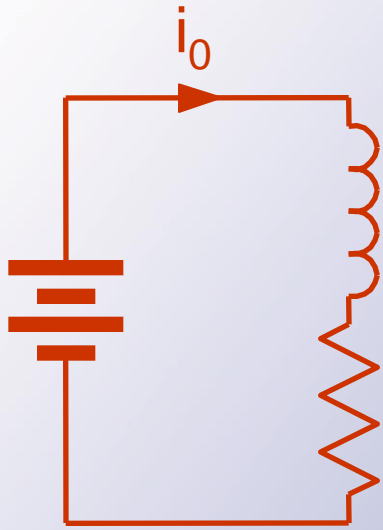
We can represent the periodic waveform with a Fourier series.

Equivalent Sources

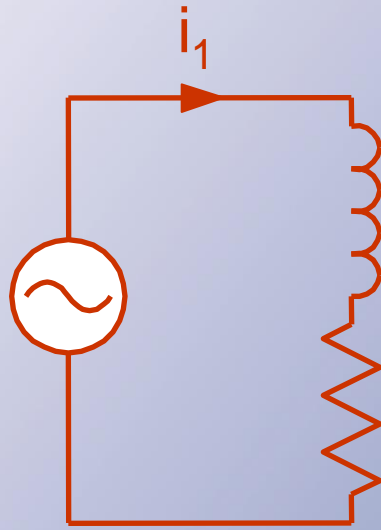


i is the sum of the contributions from each of the sources. We can break up the circuit.

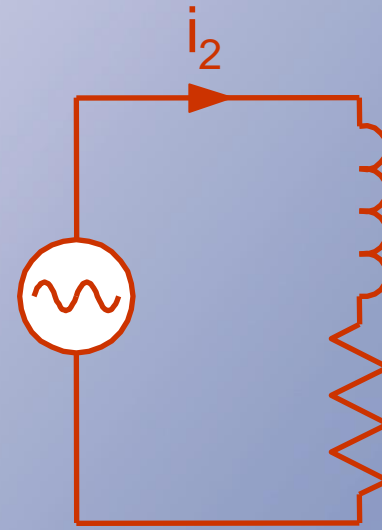
Equivalent Sources



DC



1st harmonic



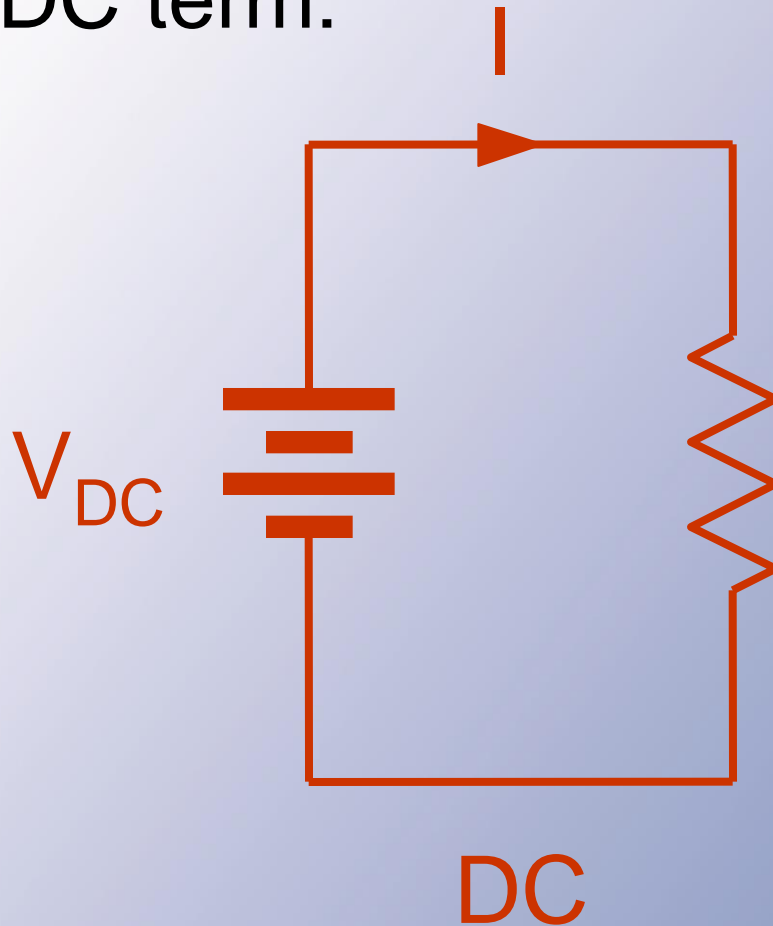
2nd harmonic

.....

$$i = \sum_{n=0}^{\infty} i_n$$

Equivalent Sources

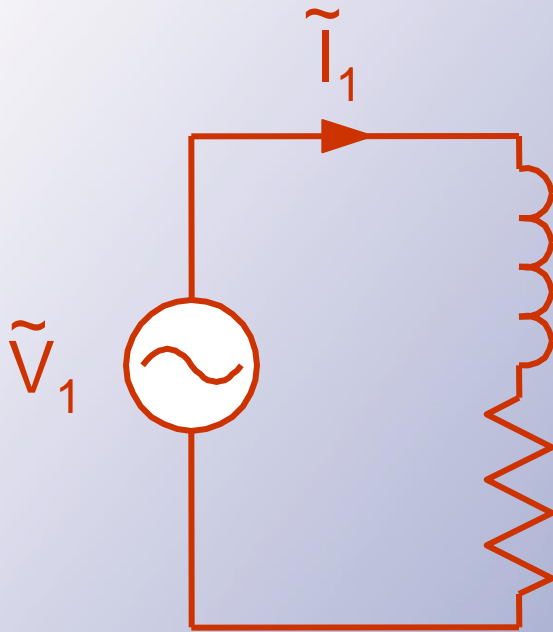
DC term:



$$I_{dc} = \frac{V_{dc}}{R}$$

Equivalent Sources

AC terms, based on phasor analysis.



$$\tilde{I}_1 = \frac{\tilde{V}_1}{R + j\omega_1 L}$$

Want low ripple
→ e.g., want $|\tilde{I}_1|$ low

Usually, Fourier terms decrease in amplitude as $1/n$. The fundamental is the largest.

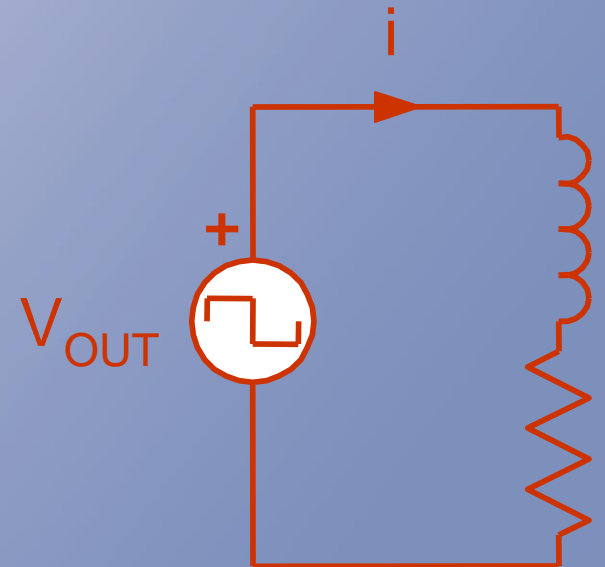
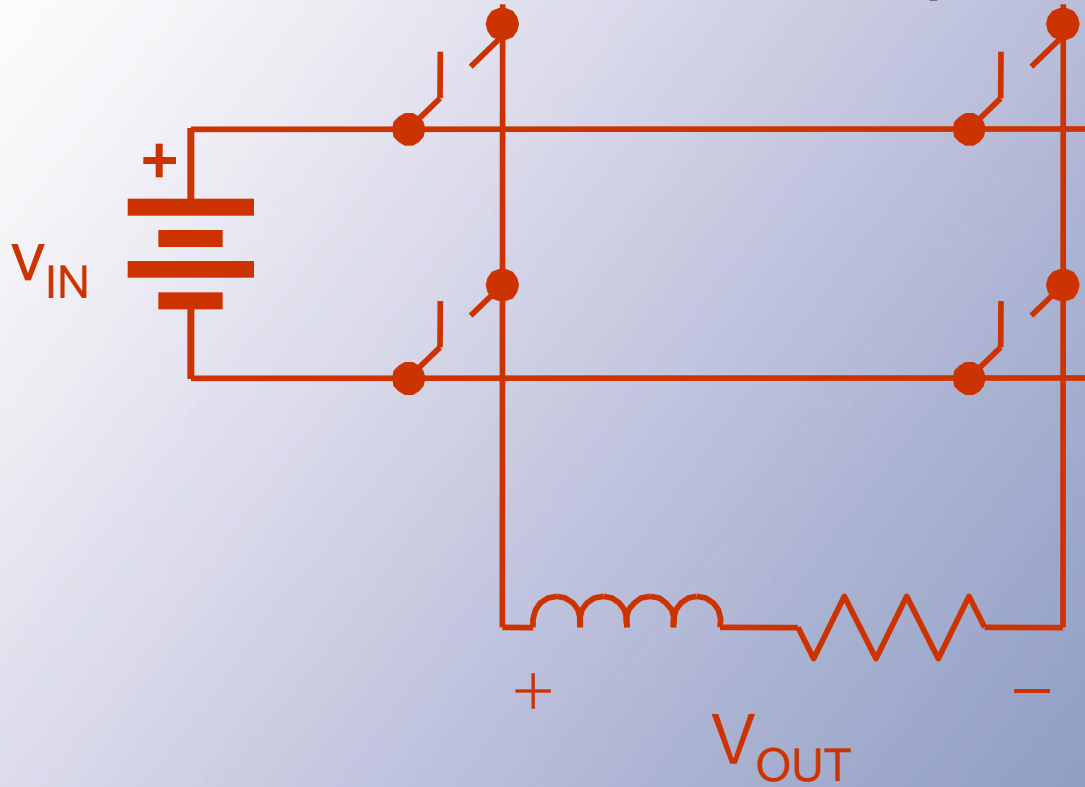
START Power Filtering

- Filters (or interfaces) for converters have needs distinct from those in signal applications.
- Filters must be lossless, and impedances of sources and loads are unknown.

Power Filtering

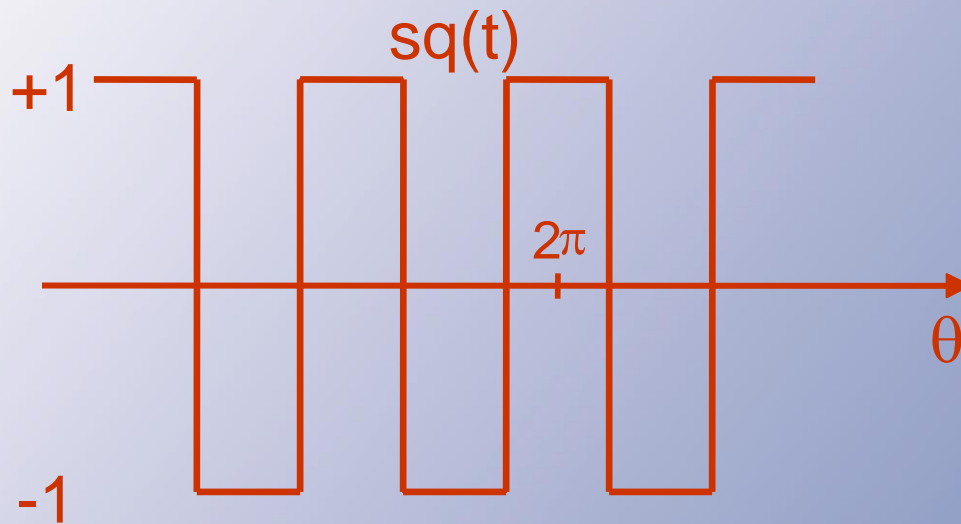
- Two common methods of analysis
 - Equivalent sources
 - “Ideal action” assumption

Filter Examples



Filter Examples

$$V_{\text{OUT}} = V_{\text{in}} \text{sq}(t)$$



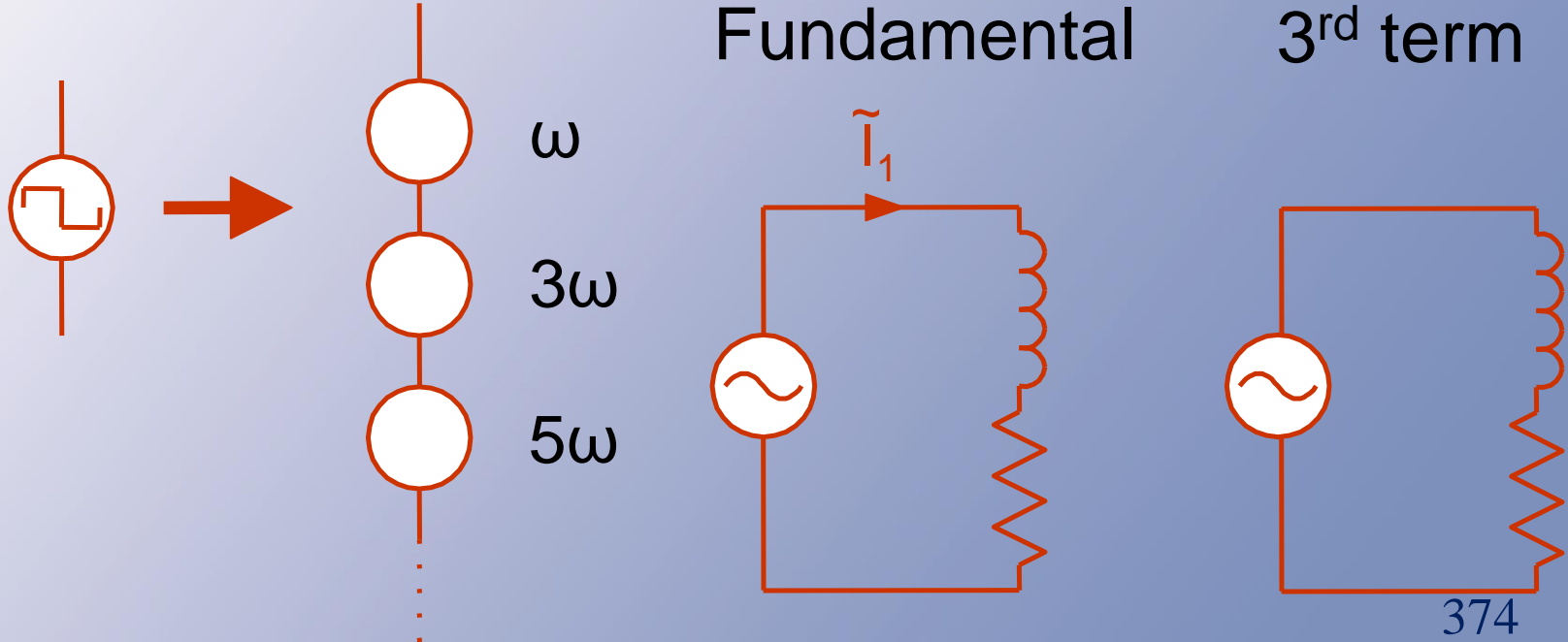
$$f = 100\text{HZ}$$

$$\text{sq}(t) = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{\sin\left(\frac{n\pi}{2}\right)}{n} \cos(n\omega t)$$

Filter Examples

$$v_{\text{out}}(t) = \frac{4V_{\text{in}}}{\pi} \sum_{n=1}^{\infty} \frac{\sin\left(\frac{n\pi}{2}\right)}{n} \cos(n\omega t)$$

$$\omega = 2\pi 100 \text{ rad/s}$$



Filter Examples

Look at examples based on the equivalent source method

Ideal Action Assumption

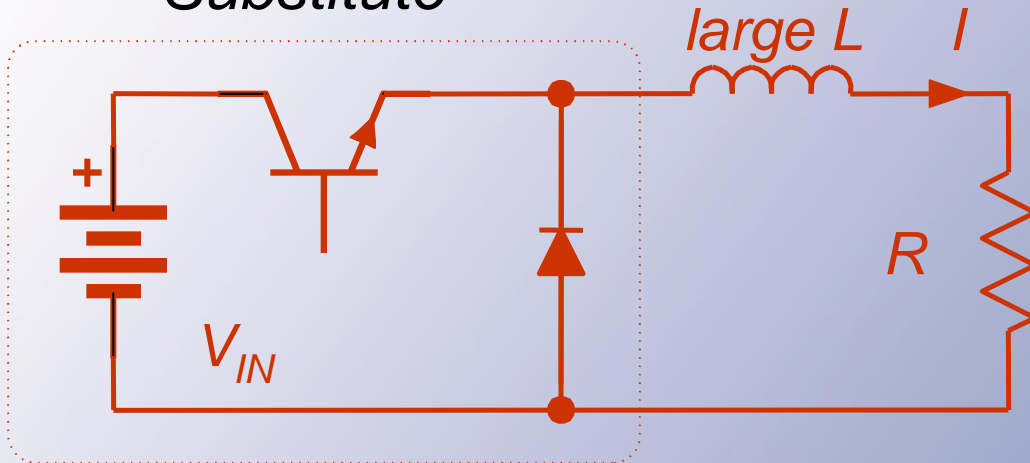
- In a power converter, we know what a filter is trying to achieve.
- Examples: low-ripple dc, ideal ac sine wave, etc.
- In general: give a large *wanted* component and small *unwanted* components.

Ideal Action Assumption

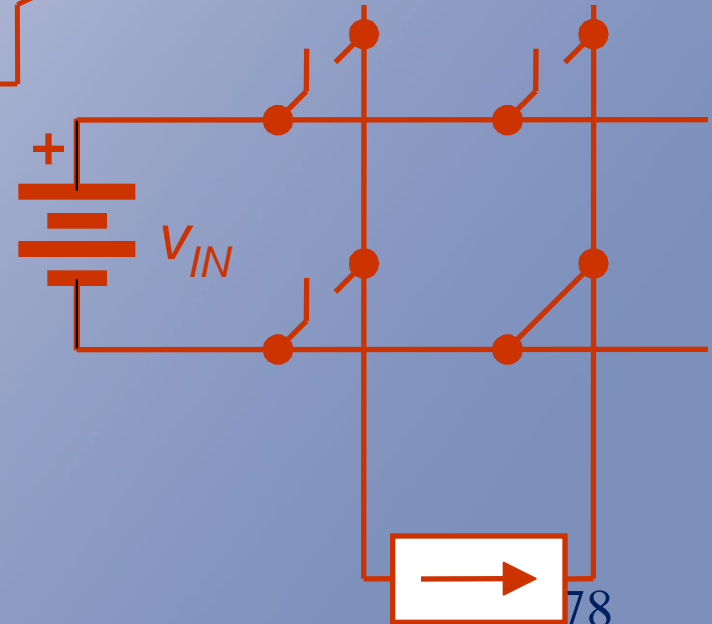
- If the filter is well-designed, it ought to work.
- If it works, we know its output.
- Now, use the “known” output with the known input to compute values.

Filter Examples

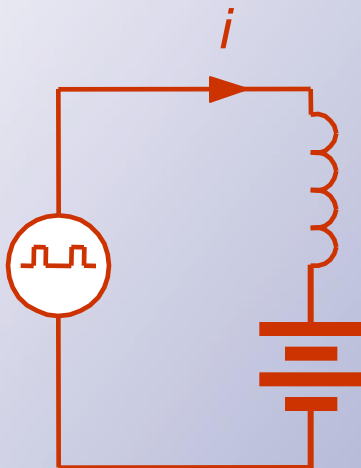
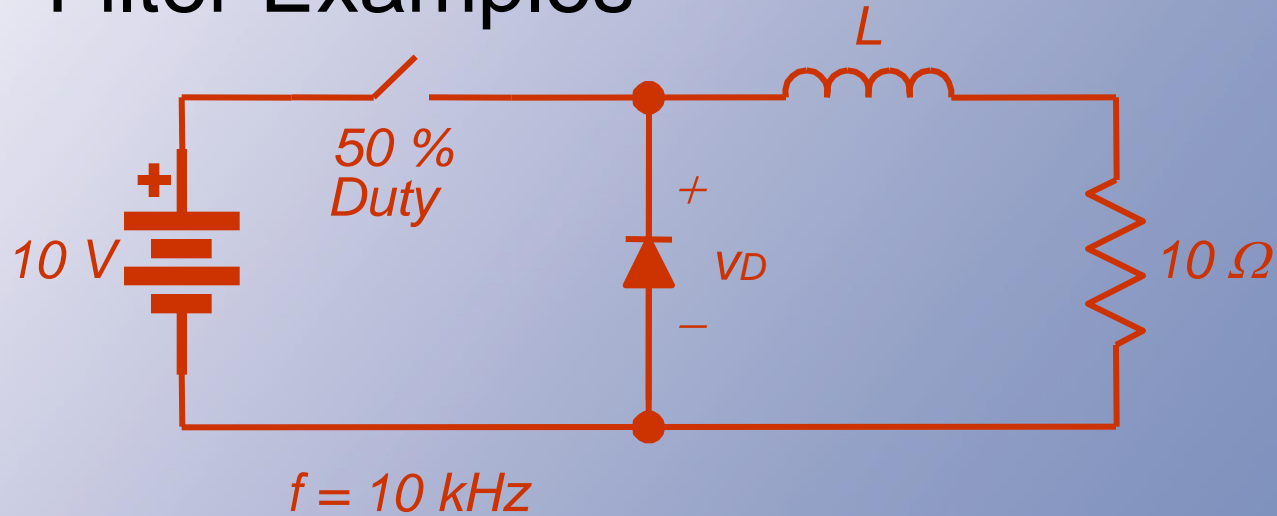
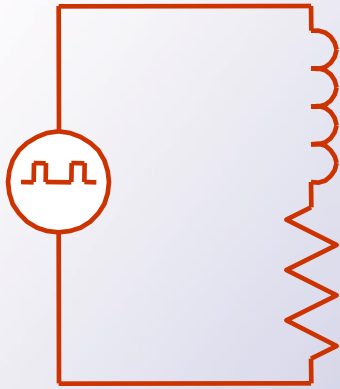
Substitute



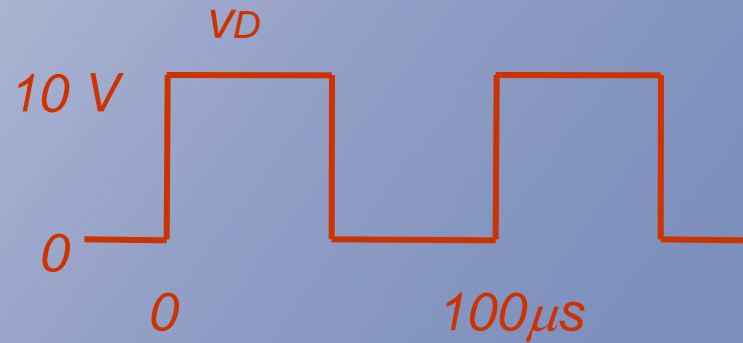
If L is large, then I is just dc.



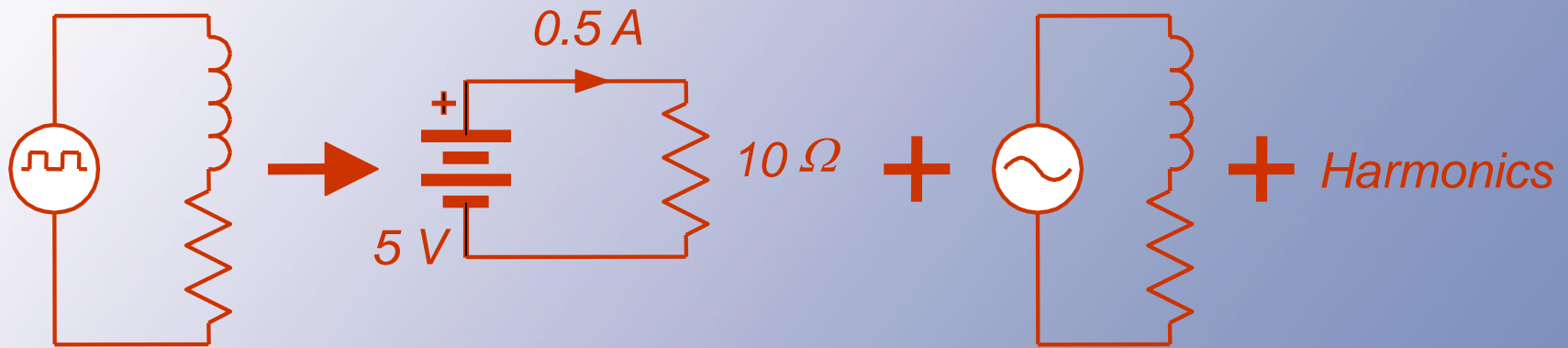
Filter Examples



Ideal action assumption



Filter Examples



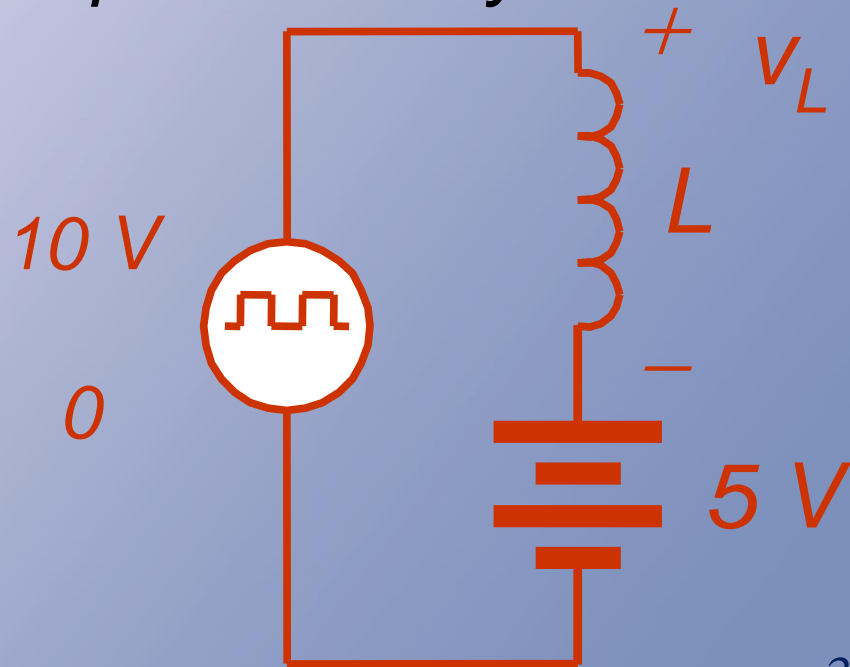
Choose L to make $|\tilde{I}_1| < \text{Limit}$.
Too much work!

Filter Examples

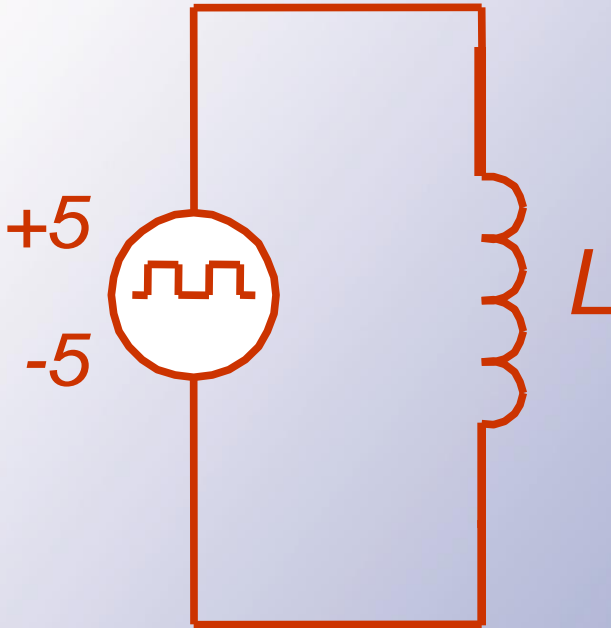
If L is large and the circuit works, the inductor current is almost constant and so is the voltage across the load resistor.

This voltage can be represented by a constant voltage source.

Switch on: $V_L = 5\text{ V}$
Switch off: $V_L = -5\text{ V}$



Filter Examples



$$V_L = L di/dt$$

$$\text{If } V_L = 5V = L di/dt$$

$$\frac{5V}{L} = di/dt$$

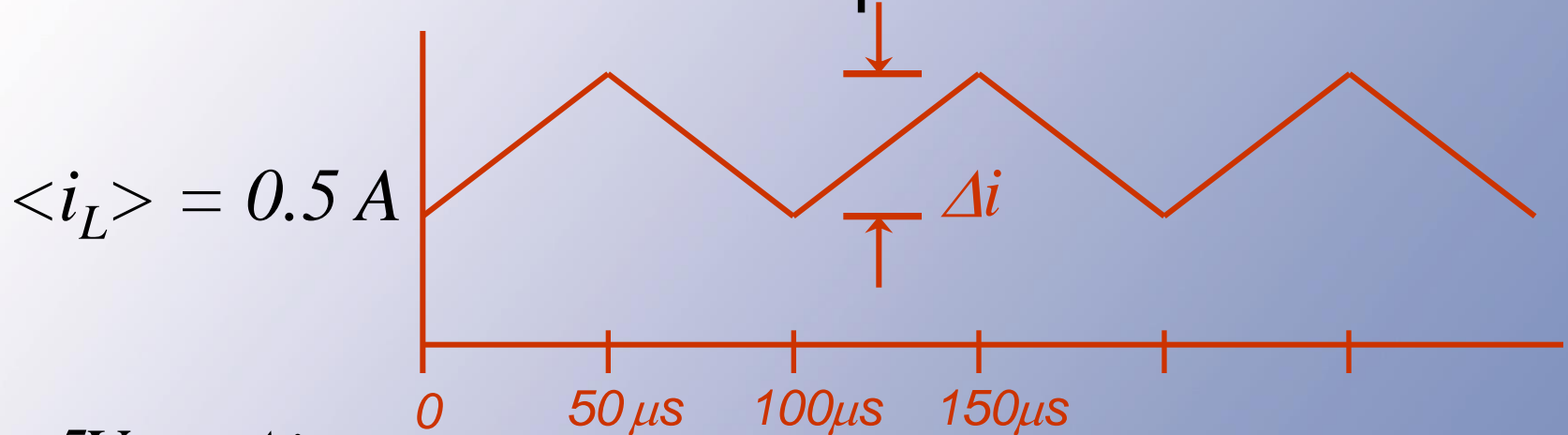
$$= \frac{\Delta i}{\Delta t}$$

$$\text{If } V_L = -5V = L di/dt$$

$$\frac{-5V}{L} = di/dt$$

$$= \frac{-\Delta i}{\Delta t}$$

Filter Examples



$$\frac{5V}{L} = \frac{\Delta i}{\Delta t}$$

$$= \frac{\Delta i}{50 \mu s}$$

Choose L to make $\Delta i = 0.005 A$

$$\Delta i = \frac{5V}{L} \times 50 \mu s$$

Filter Examples

$$0.005 \text{ A} = \frac{5\text{V}}{L} \times 50 \times 10^{-6}$$

$$L = \frac{250 \times 10^{-6}}{5 \times 10^{-3}}$$

$$L = 0.005 \text{ H}$$

$L \geq 5 \text{ mH}$ makes $\Delta i \leq 0.005 \text{ A}$

Results and Comments

- Since we know the objective of our filters, it is reasonable to design them based on the assumption that the objective is met!
- This simple expedient is a very effective simplifying step.

Results and Comments

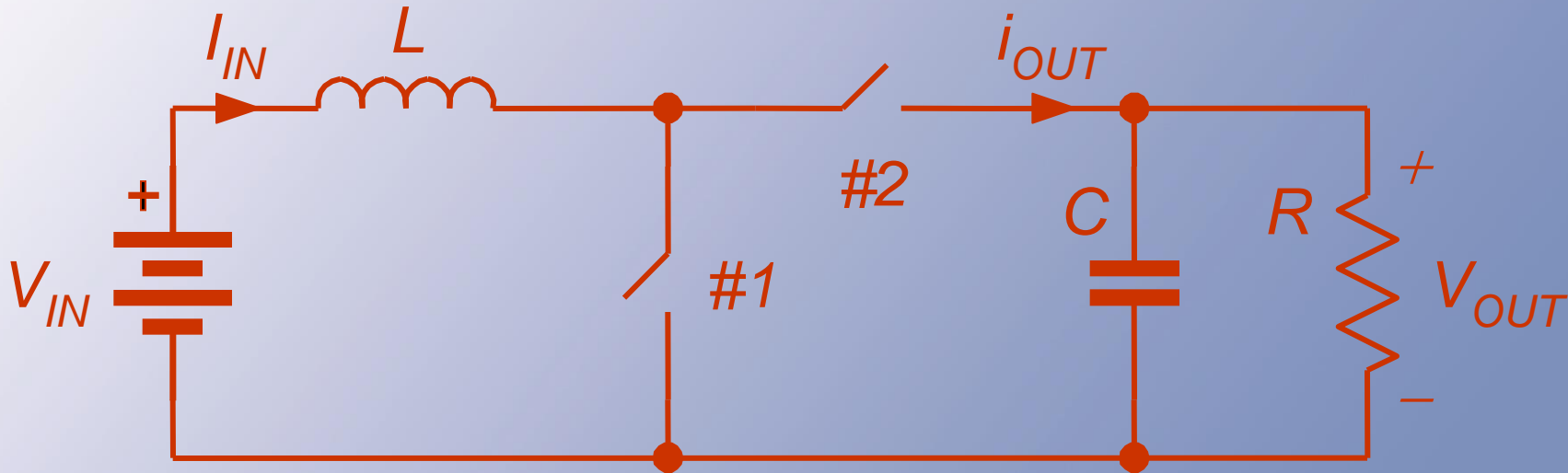
- The *ideal action assumption* works better than one might expect.
- We will analyze this as we build up converter designs.

Summary So Far

- We can analyze the quality of a converter output.
- **Equivalent sources** give us a way to deal with the **interface problem**.
- The **ideal action assumption** helps considerably with **design**.

Filter Example

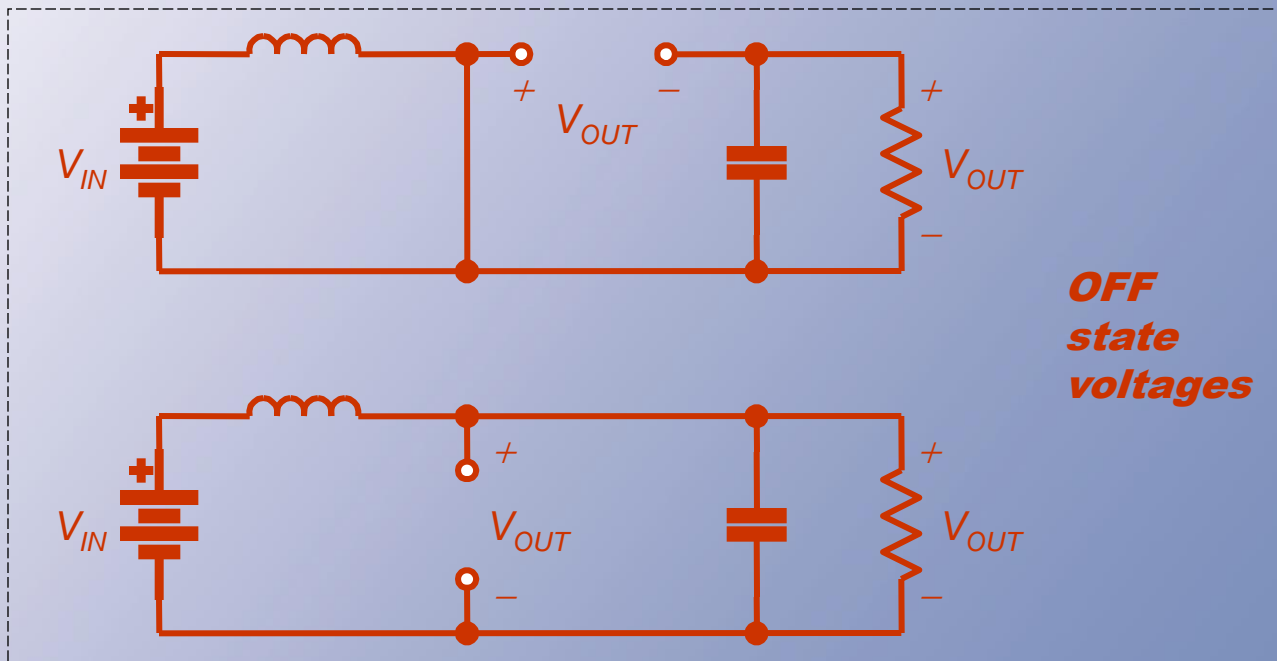
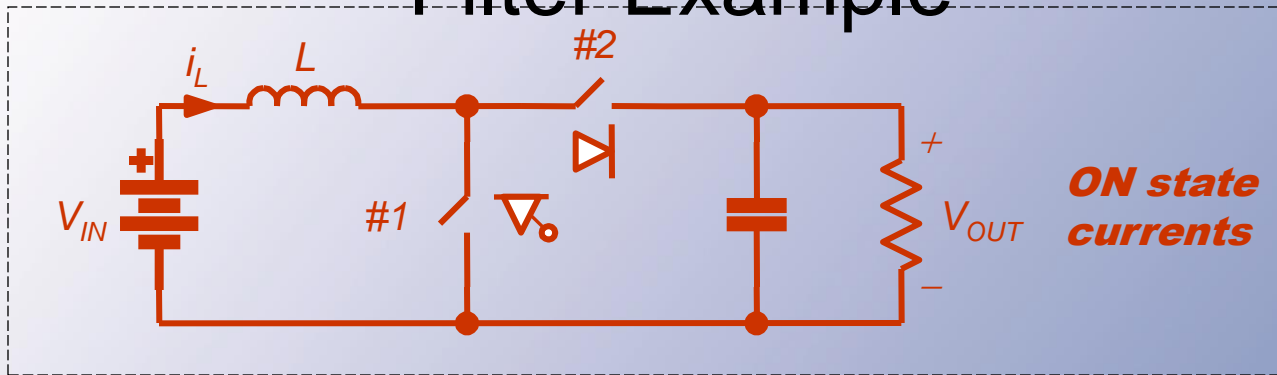
- Consider a converter, shown, with switch #1 duty ratio at 3/4.



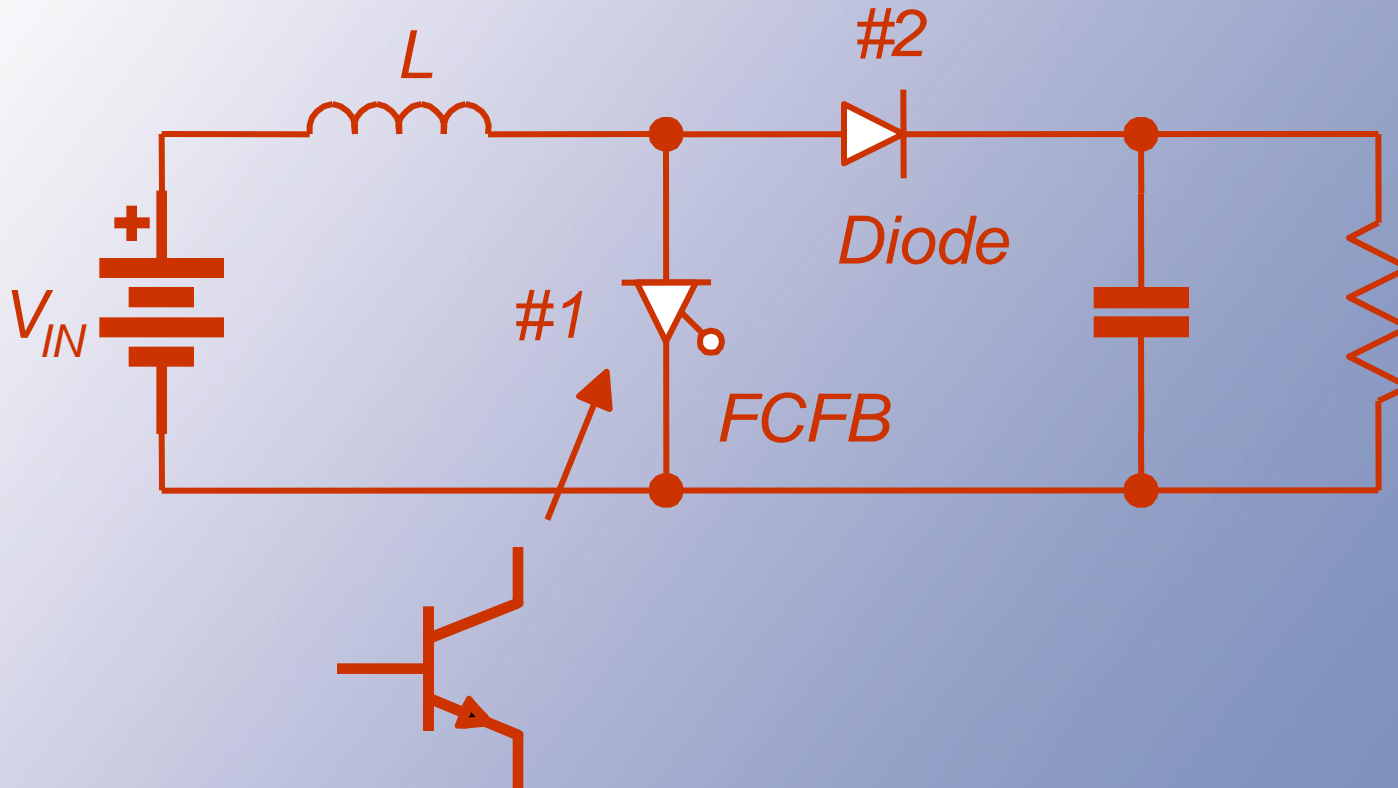
Filter Example

- Let the switching frequency be 200 kHz, $L = 1 \text{ mH}$, $C = 10 \text{ } \mu\text{F}$, $R = 10 \text{ } \Omega$, $V_{\text{in}} = 5 \text{ V}$.
- By KVL and KCL, the switches need to alternate.
- We can determine the device types.

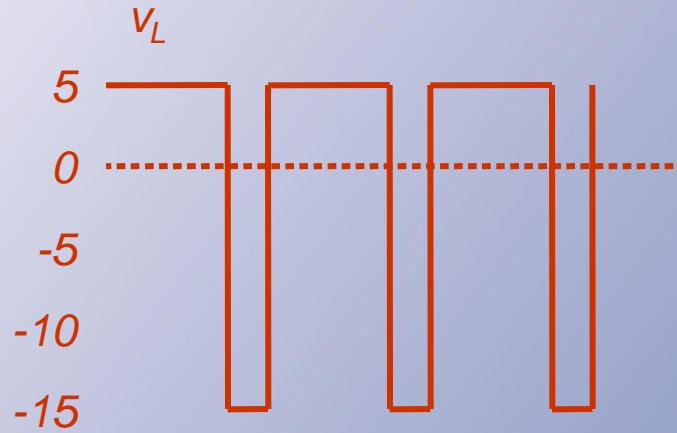
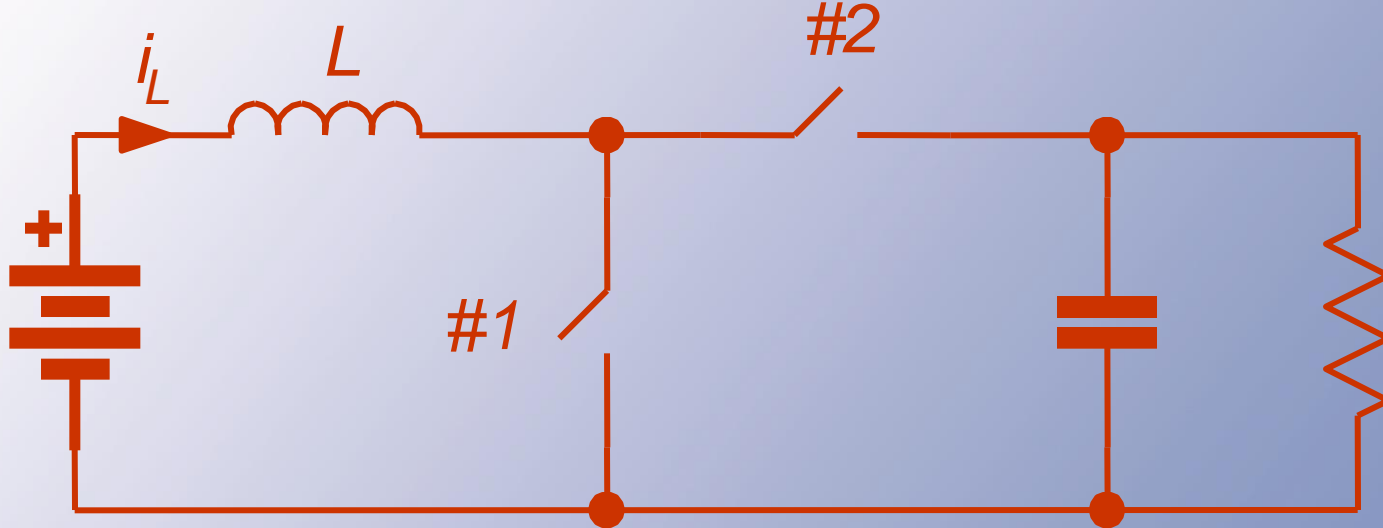
Filter Example



Filter Example



Load Current



$$\langle v_L \rangle = 0$$



Energy Balance

- With switch #1 on, the input energy to the inductor is $(V_{in})(i_L)(3T/4)$. With switch #2 on, the input is $(V_{in} - V_{out})(i_L)(T/4)$.
- The total must be zero. This requires $V_{out} = 4 V_{in} = 20 \text{ V}$.

Load Current

- The load current is 2 A, and the load power is 40 W.
- The average input current must be $(40 \text{ W}) / (5 \text{ V}) = 8 \text{ A}$. This is i_L .

Current Ripple

- If the inductor and capacitor are large (we will check this), then i_L and V_{out} are nearly constant.
- The inductor sees 5 V when #1 is on, so its current increases for 3.75 μ s.

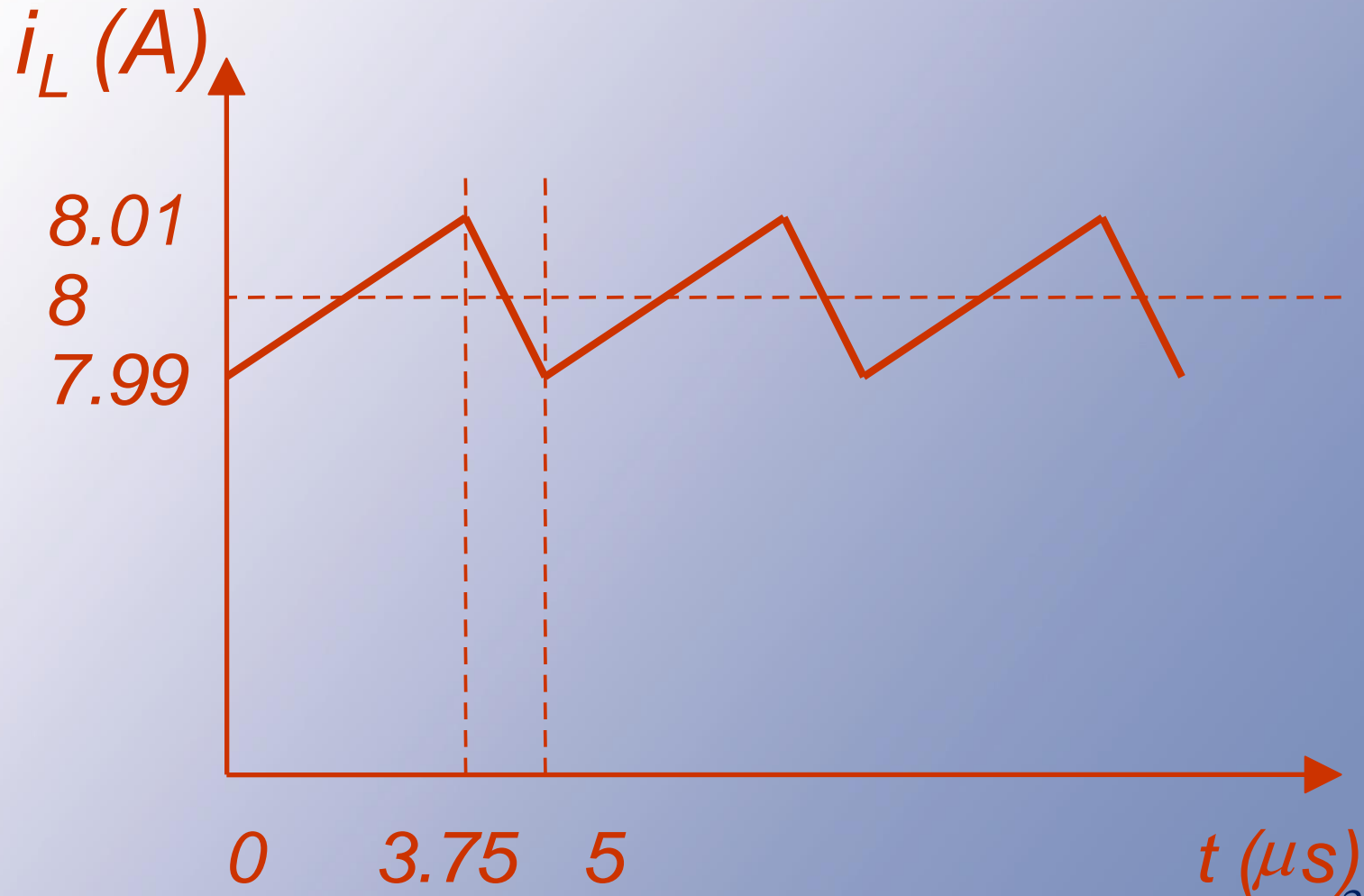
Current Ripple

- The inductor sees $5\text{ V} - 20\text{ V} = -15\text{ V}$ when switch #1 is off, and the current falls for $1.25\text{ }\mu\text{s}$.
- During the rise, $v_L = 5\text{ V} = L\text{ di}/\text{dt}$, but the rise is linear over $3.75\text{ }\mu\text{s}$, so $(5\text{ V})/L = \Delta i/\Delta t$, $\Delta t = 3.75\text{ }\mu\text{s}$.

Current ripple

- With a 1 mH inductor, this means
 $\Delta i = (5 \text{ V})(3.75 \text{ us}) / (1 \text{ mH}),$
 $\Delta i = 0.0188 \text{ A}.$
- This is less than 0.25% of i_L .
- Check the current fall. Does it match?
Why?

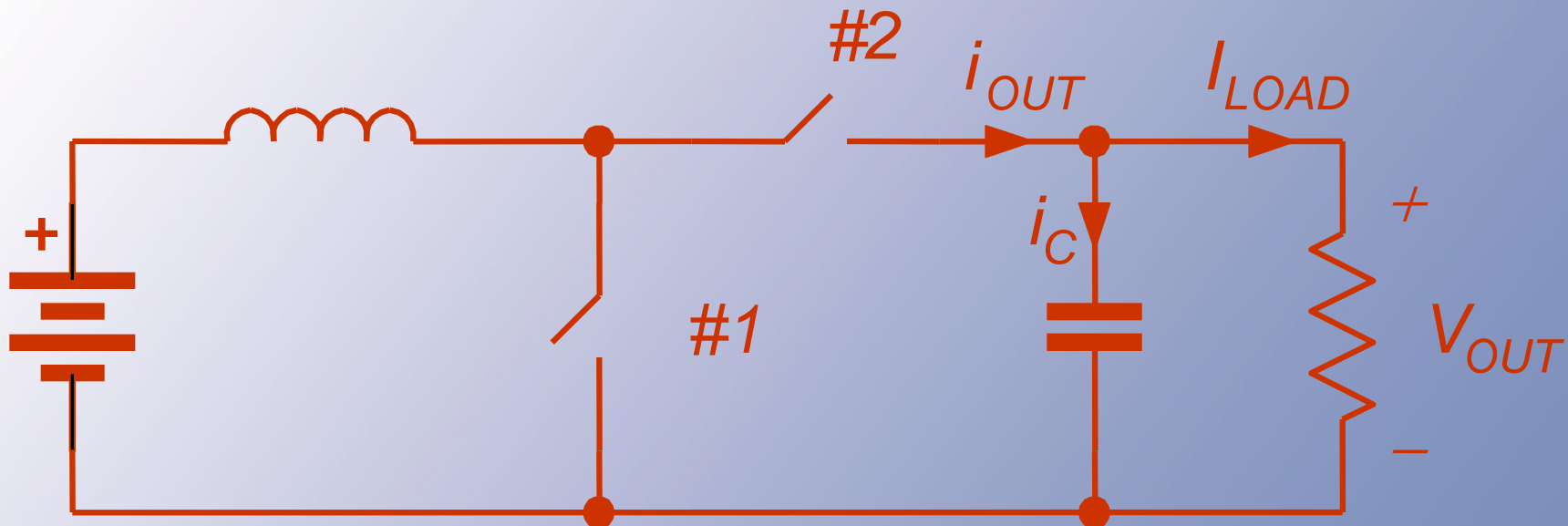
Current ripple



Voltage Ripple

- We can do the same thing to find ripple on the output capacitor.
- The capacitor current is known:
With switch #2 off, the resistor draws out 2 A. With switch #2 on, the current is $8\text{ A} - 2\text{ A} = 6\text{ A}$.

Voltage Ripple



- i_C is fully determined.
- #2 off : $i_C = -2 \text{ A}$ v_C decreases
- #2 on : $i_C = i_L - 2 = 8 - 2 = 6 \text{ A}$ v_C increases

Voltage Ripple

- Thus $i_C = 6 \text{ A}$ for 1.25 us , and -2 A for 3.75 us .
- Since $i_C = C \, dv/dt$ gives linear voltage ramps, the voltage rises when $i_C = 6 \text{ A}$:
 $(6 \text{ A})/C = \Delta v/\Delta t$.
- The time involved is 1.25 us .

Voltage Ripple

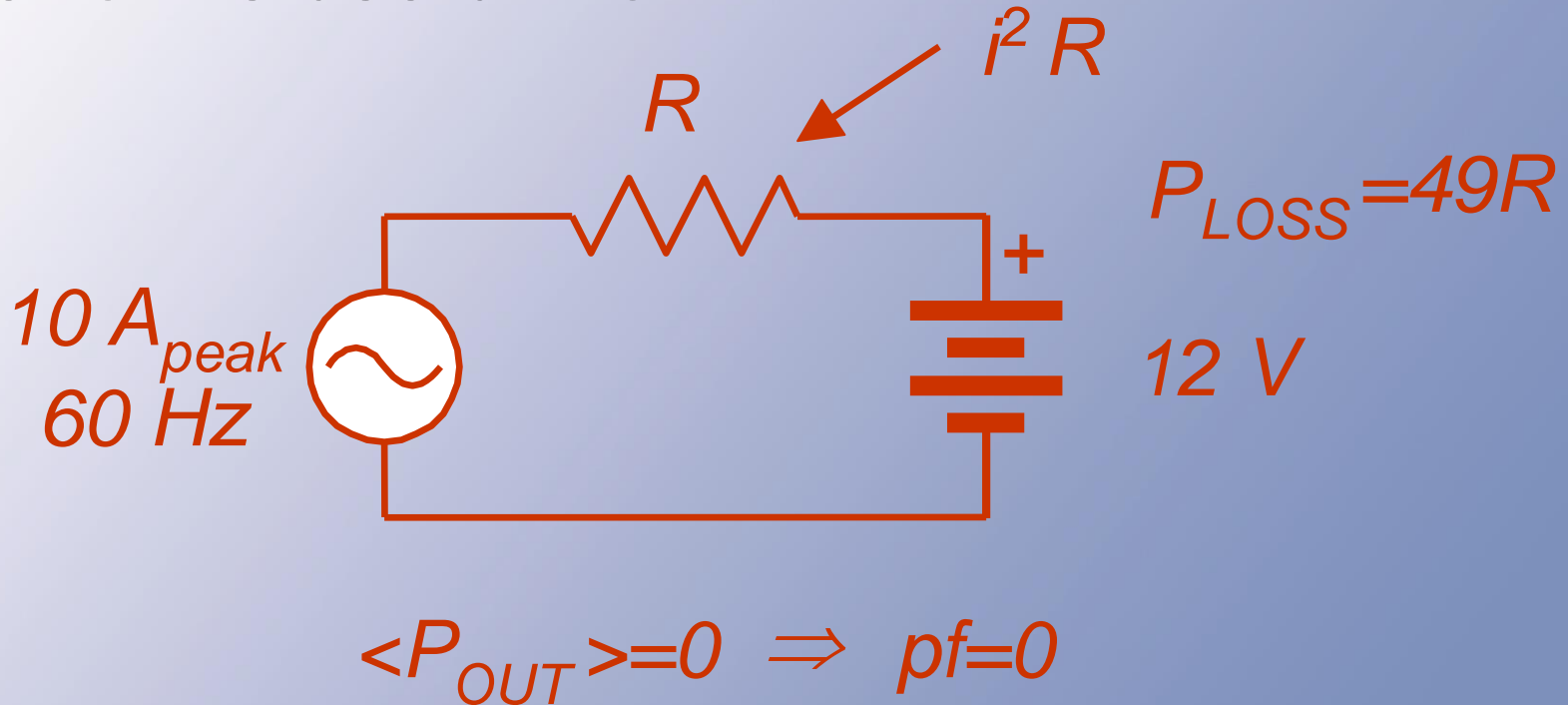
- $(6 \text{ A})(1.25 \text{ us})/(10 \text{ uF}) = \Delta v = 0.75 \text{ V}$.
- This is 3.75% of the 20 V dc level.
- Not perfect, but still very nearly constant.
- Thus with switching frequency of 200 kHz, $L = 1 \text{ mH}$, $C = 10 \text{ }\mu\text{F}$, $R = 10 \text{ }\Omega$, $V_{\text{in}} = 5 \text{ V}$, we get 20 V out and 3.75% peak-to-peak output ripple.

Power Factor

- A conventional measure in utility systems is *power factor* -- the fraction of energy flow that does useful work.
- Recall that cross-frequency terms do not contribute $\langle P \rangle$.
- But, the cross terms *do* require current and voltage.
- The extra current means extra I^2R loss, and should be avoided is possible.

Power Factor

Capture fraction of energy flow that performs useful work.



Power Factor

- Power factor is defined by

$$pf = \frac{\langle P \rangle}{V_{RMS} I_{RMS}} \leq 1$$

- Ideally, this is 1. When harmonics or phase shifts are present, it is less than 1.
- pf can be less than 1 even in a linear circuit, but it is never greater than 1.

Power Factor Example



$$\langle P \rangle = 0 \Rightarrow pf = 0$$

Two contributions to the pf : “Distortion power” and “Displacement power.” The “*displacement factor*.”

$$df = \frac{\langle P \rangle}{V_{RMS1} I_{RMS1}} = \cos(\theta_1)$$

Power Factor Issues

- pf is often divided into a phase effect at the wanted frequency (*displacement power, with a displacement factor*), and a distortion effect at unwanted frequencies.
- $pf < 1$ causes extra loss, and limits flow capabilities.

Power Factor Issues

Why do we want $\text{pf} = 1$?

1) Minimizes system loss. Maximizes “device utilization.”

2) Gives more available power.

120 V, 12 A

$\text{pf} = 1 \rightarrow 1440 \text{ W}$

$\text{pf} = 0.5 \rightarrow 720 \text{ W}$

3) Examples

Rectifiers can have $\text{pf} \sim \underline{0.3}$

1440 V \rightarrow 430 W



Dc-Dc Converters

- We would like to have a dc transformer -- a device with $P_{in} = P_{out}$ and $V_{out}/V_{in} = a$.
- Magnetic transformers cannot handle dc, but the dc transformer is still a valid concept.
- Our objective in dc-dc converter design is to approach a dc transformer as best we can.

Dc Transformers

- We would like to have a box like this, for DC.



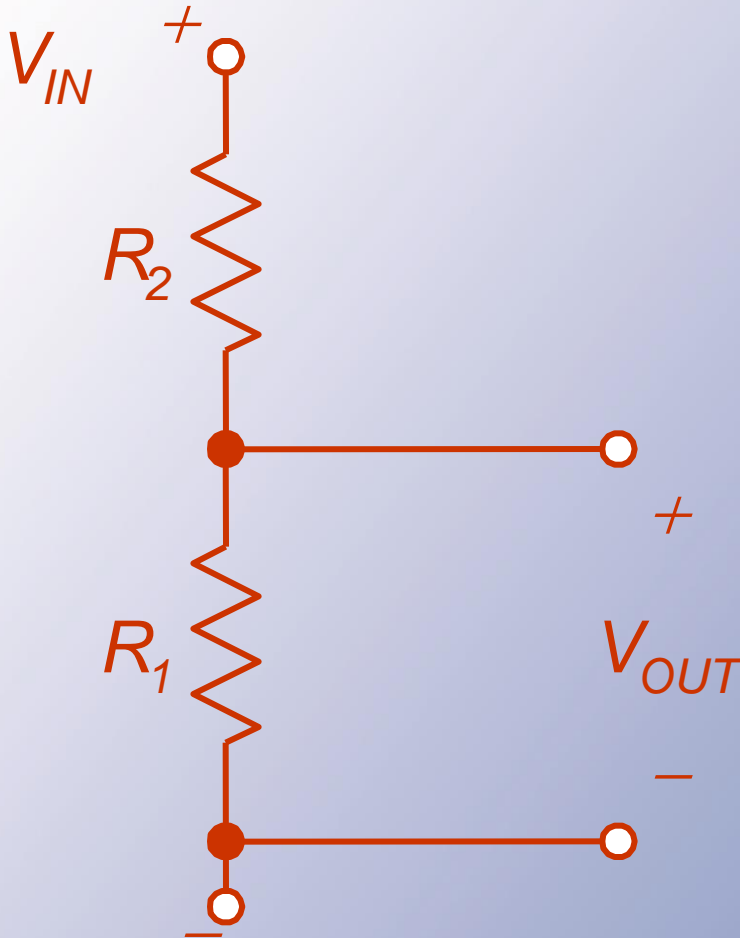
$$P_{in} = V_{in} I_{in} = P_{out} = V_{out} I_{out}$$

$$\frac{V_{out}}{V_{in}} = a \qquad \frac{I_{in}}{I_{out}} = a$$

Dividers?

- We might try a voltage divider.
- Two problems:
 - No regulation
 - Losses within the “converter”

Dividers?

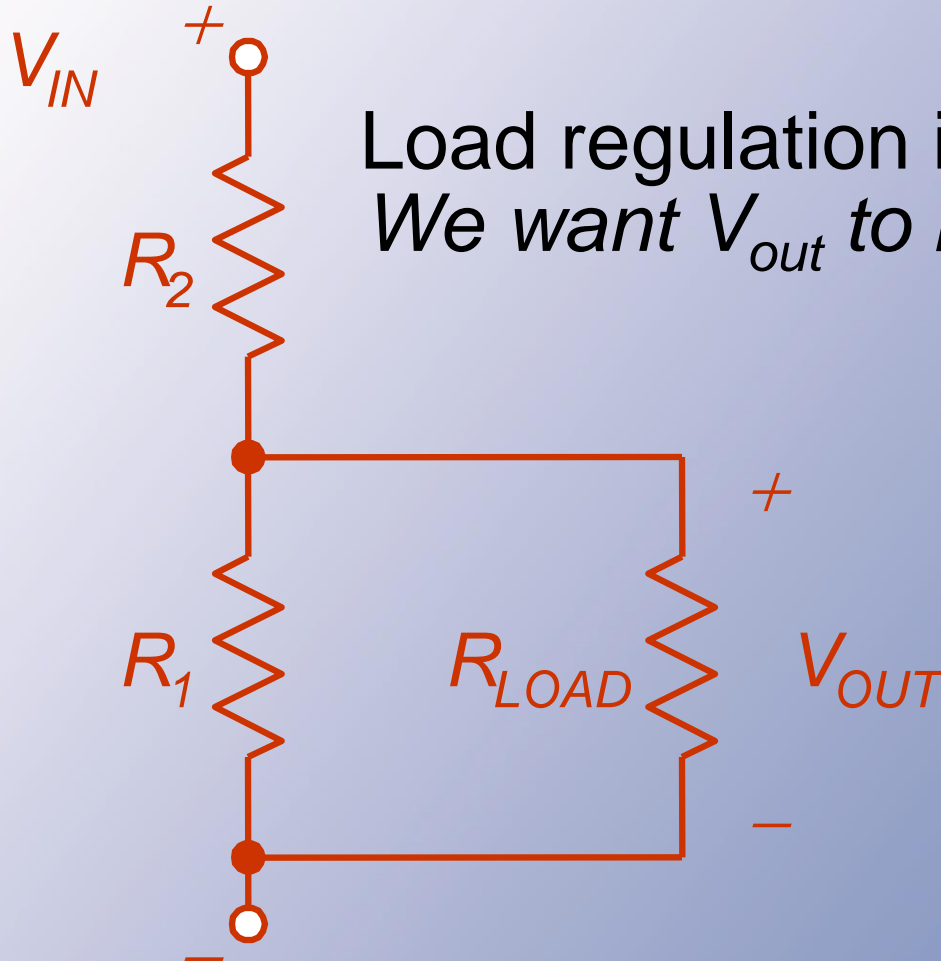


$$\eta = \frac{P_{out}}{P_{in}}$$

No load: $V_{out} = \frac{R_1}{R_1 + R_2} V_{in}$

If $P_{out} = 0$, then $\eta = 0$

Dividers?



Load regulation issue.
We want V_{out} to be almost constant.

Dividers?

- The load regulation problem can be addressed through **excess loading**:
- Make the divider input draw so much power that the load power causes minimal change.

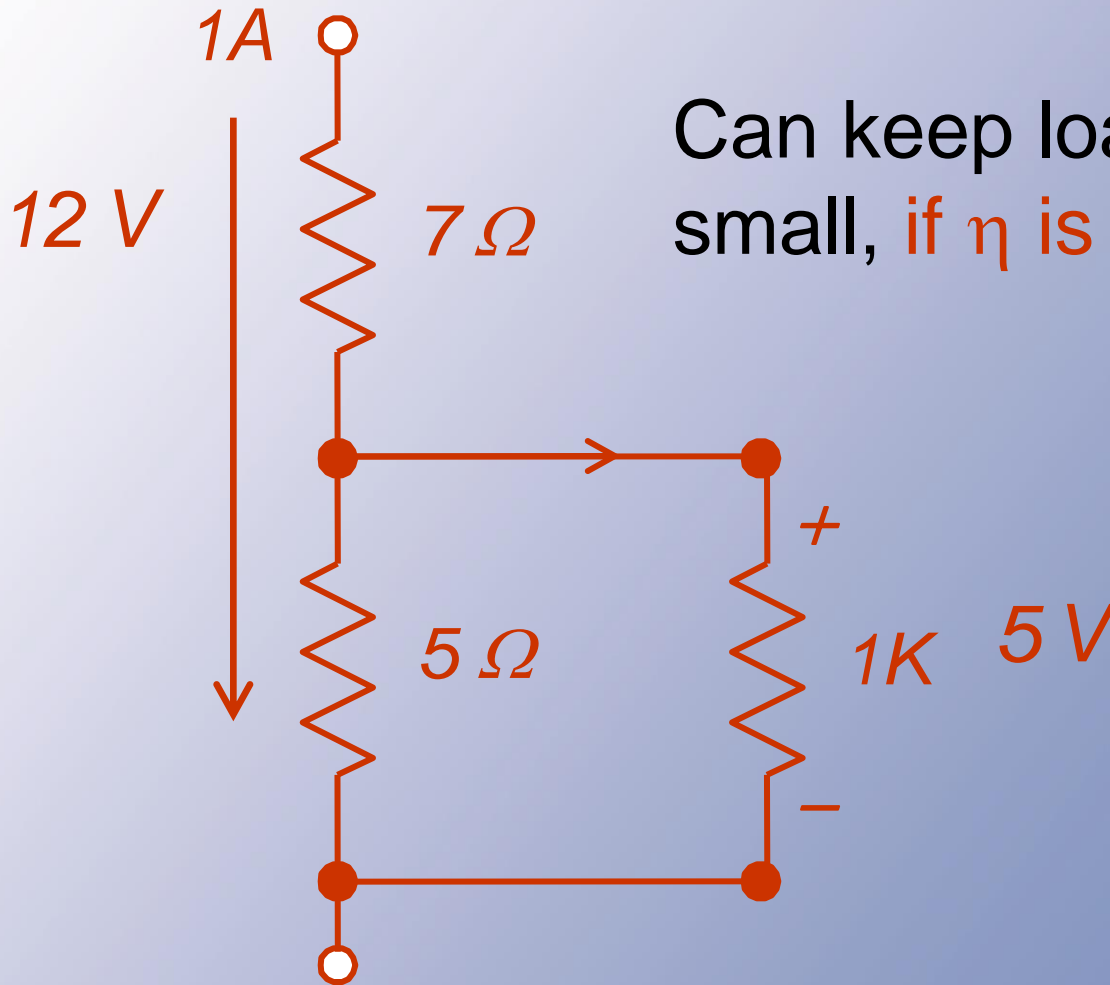
Divider Efficiency

- Instead, if somehow all output power is delivered to the load (best possible case), the efficiency is $V_{\text{out}}/V_{\text{in}}$.
- This occurs only at a single load value, if designed in advance. The design has no load regulation.
- **Reality is always worse.**

Dividers -- Conclusion

- Voltage dividers are useful for sensing applications when the load power is intended to be zero.
- A voltage divider is *not* useful for dc-dc conversion.
- It is not a power electronic circuit, since the efficiency cannot be 100%.

Sensing application



Can keep load change small, if η is low

Sensing application

$$P_{in} = V_{in} I_{in}$$

$$P_{out} = V_{out} I_{out}$$

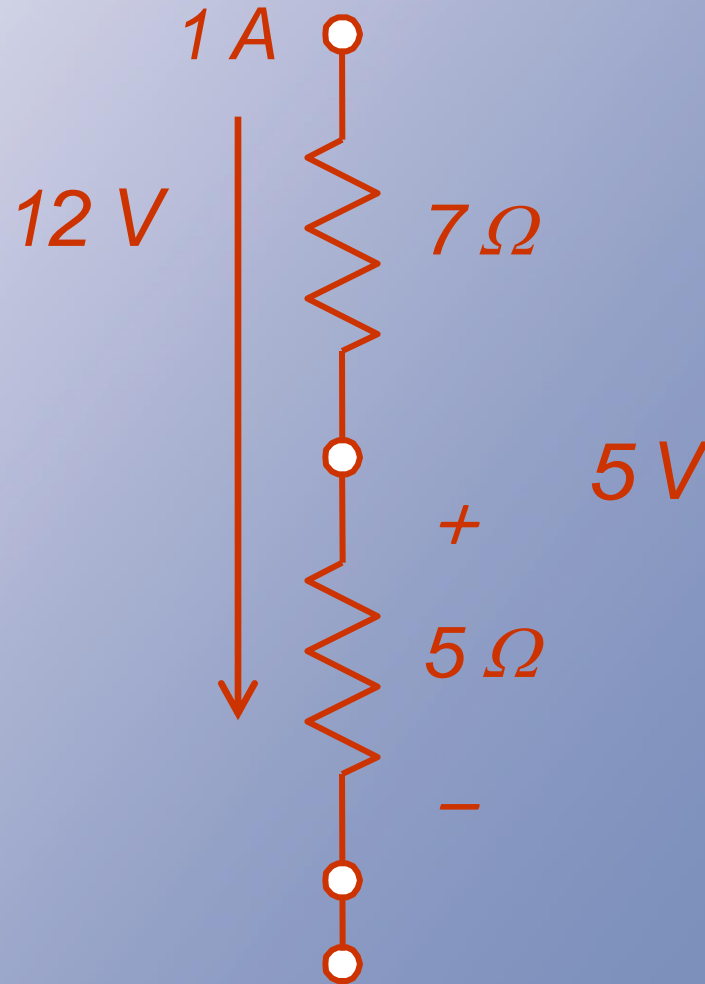
$$I_{in} = I_{out}$$

$$\frac{P_{out}}{P} = \frac{V_{out} I_{in}}{V_{in} I_{out}}$$

$$= \frac{V_{out}}{V_{in}}$$

$$= \frac{V_{out}}{V_{in}}$$

$$\eta = 5/12$$



Dc Regulators

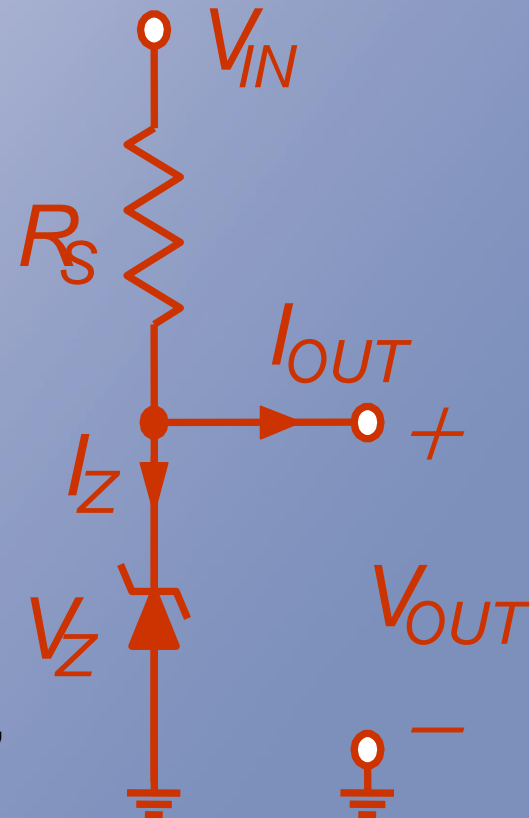
- Since a divider has no regulation, it motivates new types of circuits.
- In these types of “converters,” the output is independent (within limits) of the input and of the load.
- They perform a regulation function rather than energy conversion.
- We call them “dc regulators.”

Amplifiers

- It is also possible to use **amplifier methods** for dc-dc conversion.
- These are common, because they have **excellent regulation** properties.
- In general, **efficiency is poor**.

Shunt Regulator

- Voltage divider, 12 V to 5 V, 1 W.
 - With exact values, best efficiency is 5/12.
 - To provide regulation, the divider current path must carry more than the load current.
 - Problems: line regulation, load regulation, loss even if $P_{\text{out}} = 0$, low η .
- Shunt regulator.
 - Zener diode in place of low-side resistor.
 - Requires $I_Z > 0$.
 - For 12 V to 5 V, 1 W, $R_1 < 35 \Omega$.
 - Solves the line and load regulation challenges, but not the others.



Example

- 12 V to 5 V regulation at up to 0.2 A.
- At 0.2 A load, the input current must be at least 0.2 A to ensure $I_Z > 0$.
- This current flows through a drop of 7 V, so $R_S < 35 \Omega$.
- Try it

Example

- Test a load of 0.1 A. The input current, if the regulator works, is $(12\text{ V} - 5\text{ V}) / (35\ \Omega) = 0.2\text{ A}$. The load current is 0.1 A, so the **zener current must be 0.1 A**.
- This is wasteful, but it works.
- Useful for generating low-power reference voltages.

Example

$$\begin{aligned} P_{\text{OUT}} &= (0.1 \text{ A})(5 \text{ V}) \\ &= 0.5 \text{ W} \end{aligned}$$

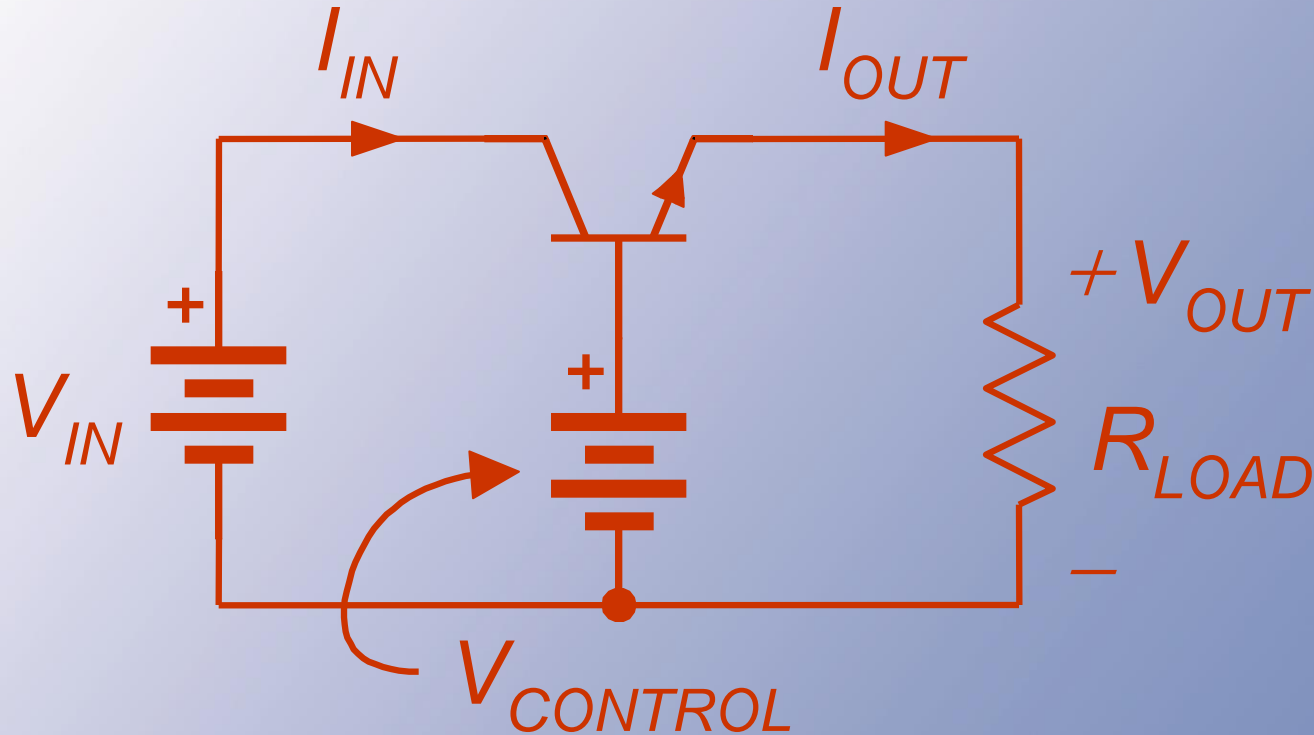
$$\begin{aligned} P_{\text{IN}} &= (12 \text{ V})(0.2 \text{ A}) \\ &= 2.4 \text{ W} \end{aligned}$$

$$\begin{aligned} \eta &= \frac{P_{\text{out}}}{P_{\text{in}}} \\ &= 20.8\% \end{aligned}$$

Series Regulator

- Instead find a series device that can provide an output that is approximately independent of the input.
- A **bipolar transistor** can do the job – in its linear operating region.
- With proper bias, the output depends on the base voltage.
- **Not** a switching method.

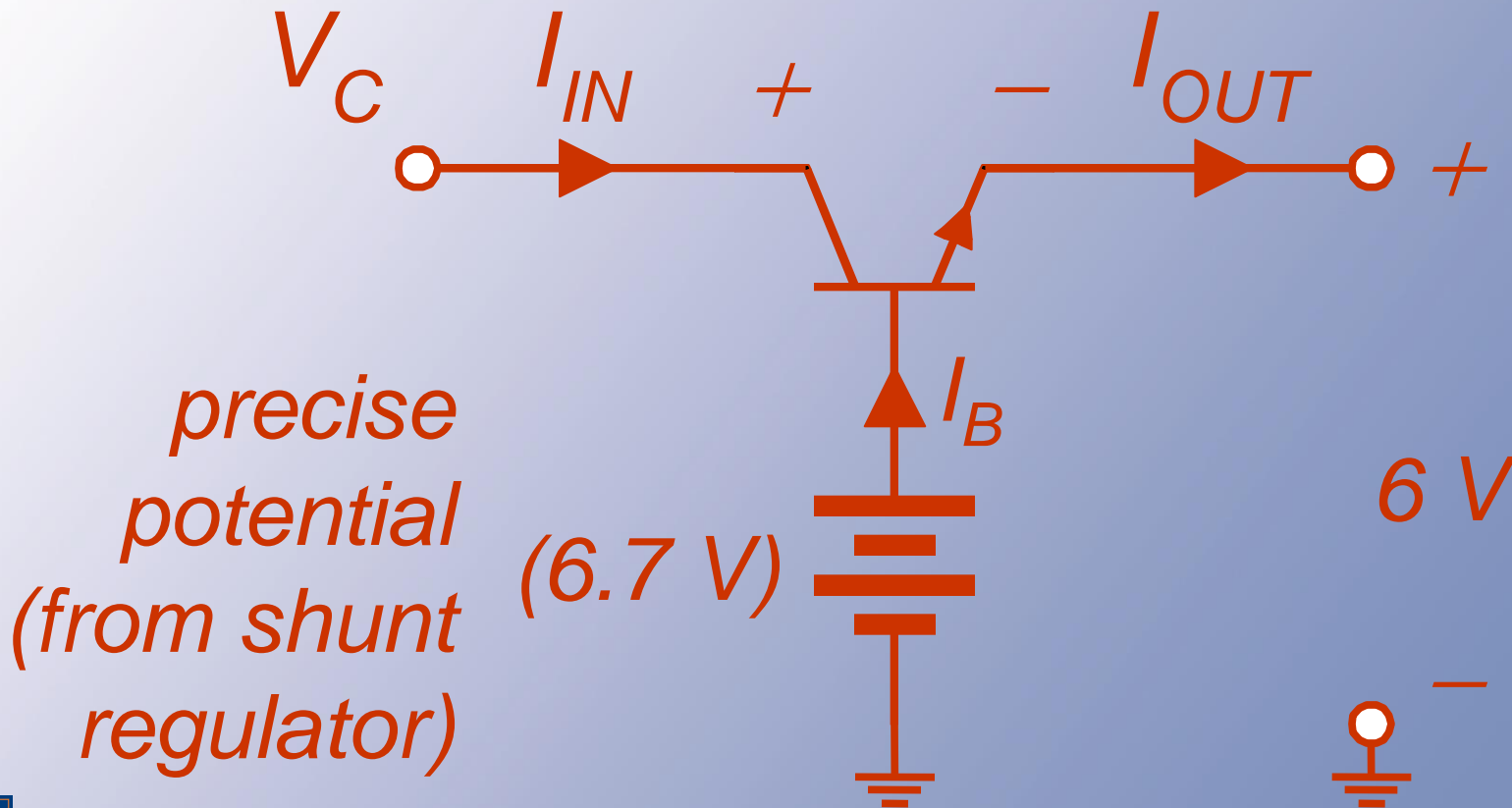
Series Pass Arrangement

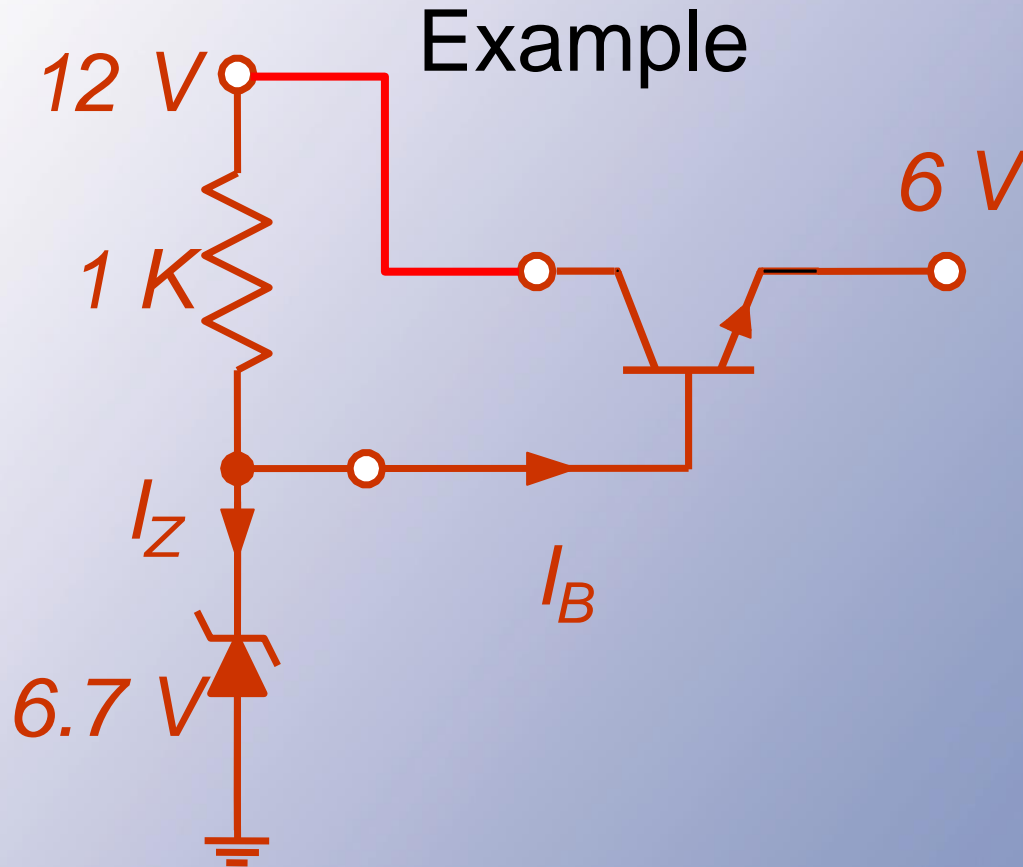


The emitter voltage follows the (low-power) base voltage.

Series Pass Arrangement

Suppose a 6 V output is needed.





Here, a shunt regulator provides the reference voltage for a series regulator.

Series Pass Arrangement

- In the bipolar case, if there is high gain, the base current is very low.
- The emitter voltage will be roughly 0.7 V below the base voltage.
- This works provided the collector input is high enough.

Series Pass Arrangement

$I_{IN} = I_C$ If I_B is small (high gain), then

$$\begin{aligned} I_{OUT} &= I_E & I_C &= I_E \\ &= I_B + I_C & I_{IN} &= I_{OUT} \end{aligned}$$

$$\eta = \frac{P_{out}}{P_{in}} = \frac{V_{out} I_{out}}{V_{in} I_{in}} = \frac{V_{out}}{V_{in}}$$

Series Pass Comments

- Common for local dc power, e.g., 12 V in, 5 V out, but extremely inefficient unless voltages are nearly the same.
- Notice that $I_{in} \approx I_{out}$.
- Best-case efficiency is V_{out}/V_{in} since current is conserved.
- Requires $V_{in} > V_{out} + \sim 2 V$

More Comments

- Although this is common, it is only acceptable when voltages are close.
- Useful example: 14 V to 12 V regulator for automotive application. Efficiency could be 86%.
- Poor example: 48 V to 5 V regulator for telephone application. Efficiency is only about 10%.

Key Advantage

- $V_{\text{out}} = V_{\text{control}} - V_{\text{be}}$ --- entirely independent of input, load, etc.
- This is a “**linear regulator**,” since V_{out} is a linear function of a control potential.

Parting Comments

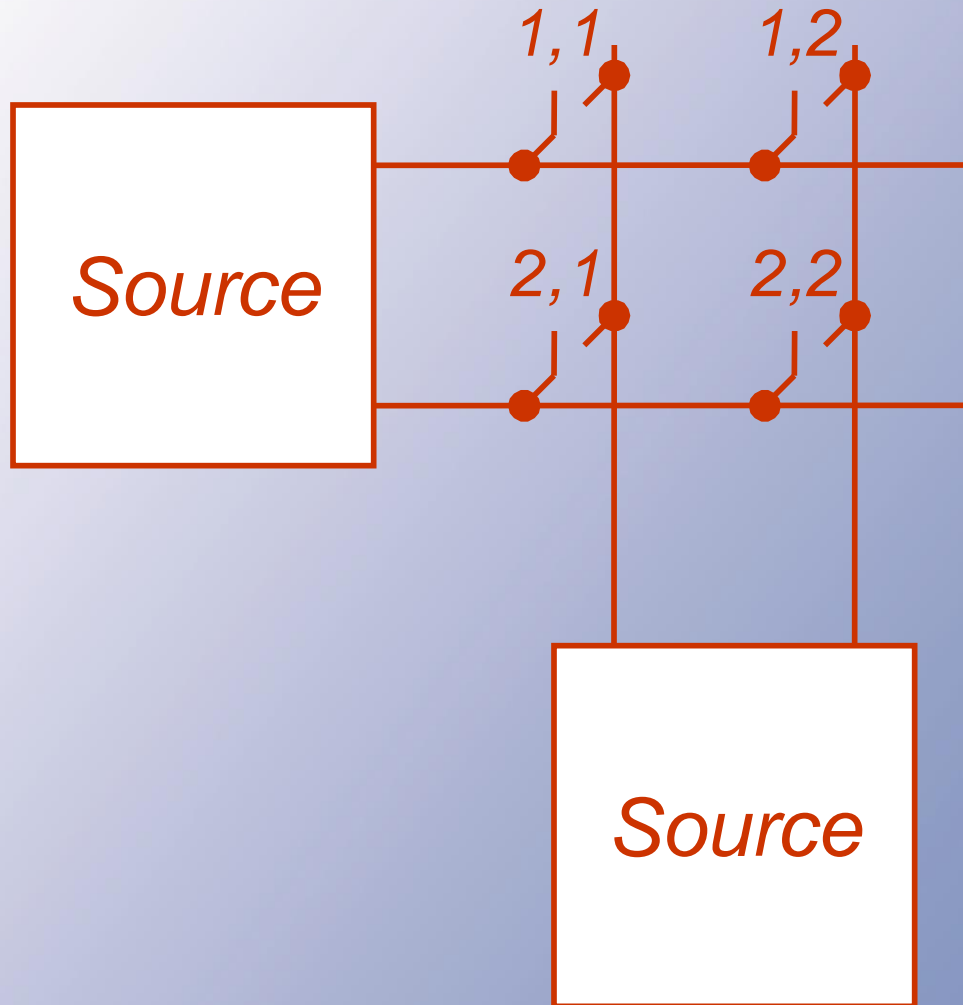
- Series linear regulators make good filters -- if we can keep the input and output close together.
- Shunt regulators provide fine fixed reference voltages but are not so useful for power.

Now, Switching

- The circuits so far cannot provide 100% efficiency. *We need switching.*
- Two possibilities of general dc-dc conversion:
 - 2×2 matrix, voltage in, current out
 - 2×2 matrix, current in, voltage out.
- These are the direct dc-dc converters.

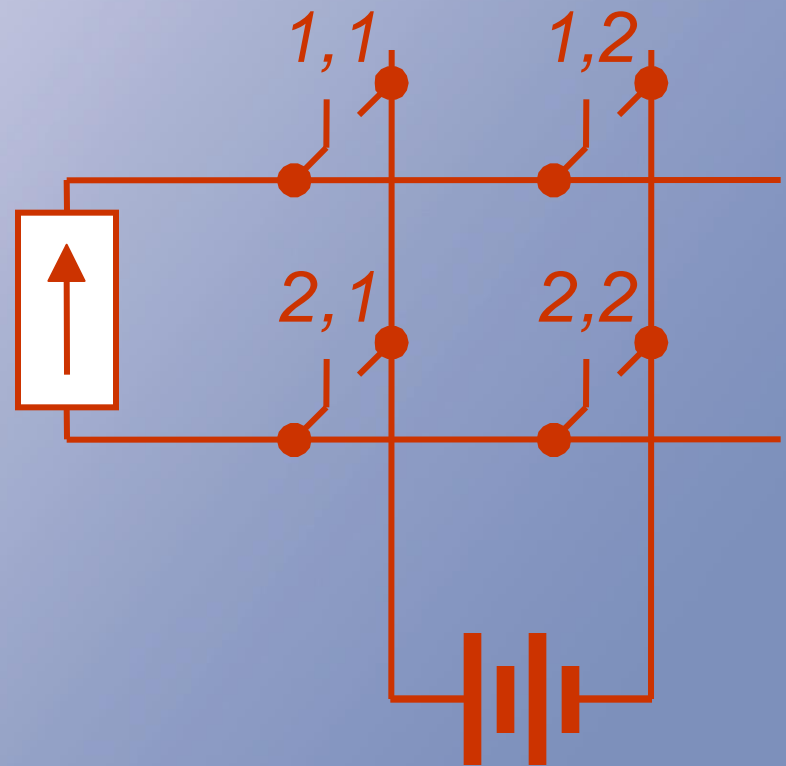
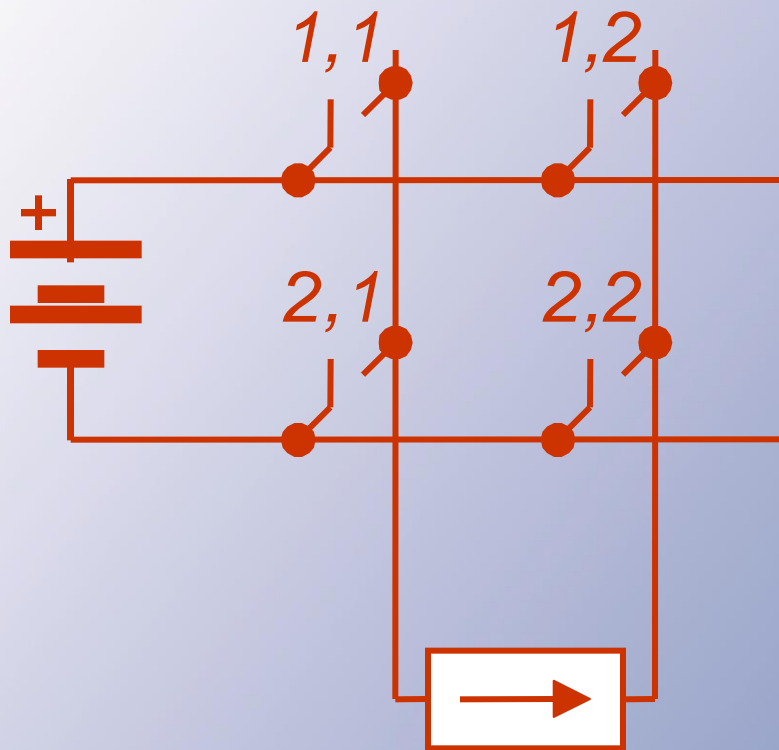


Direct DC-DC Converters

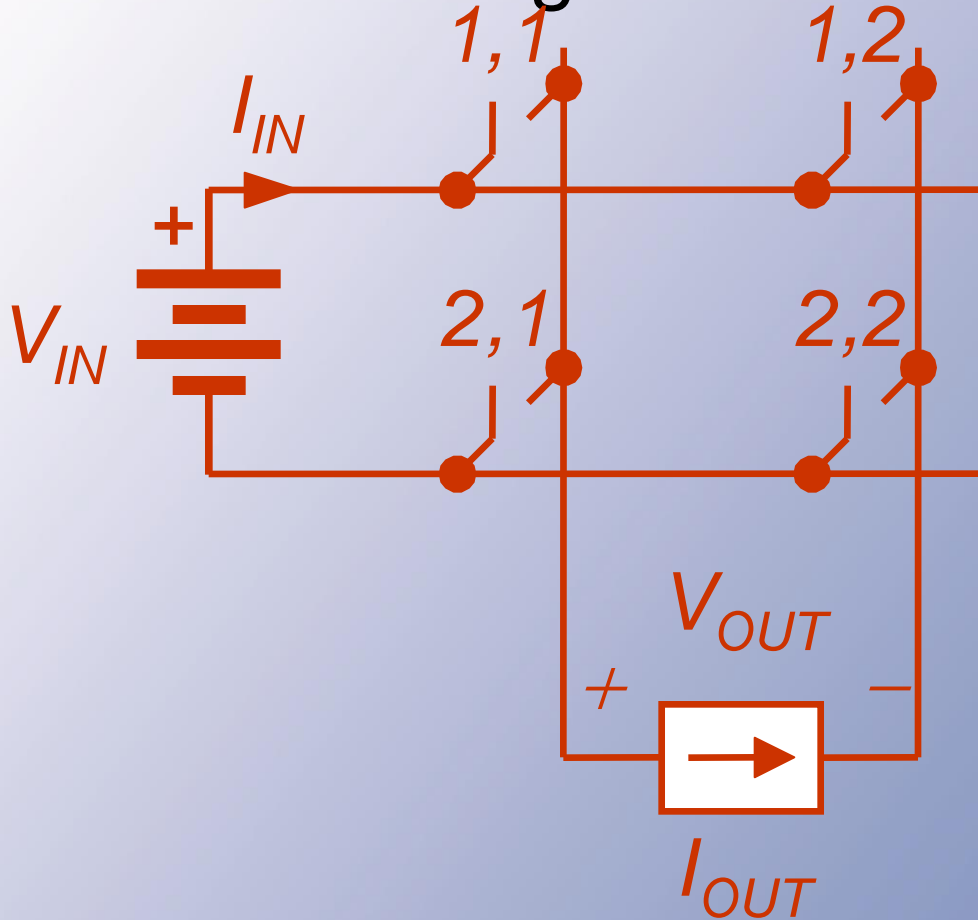


Direct DC-DC Converters

Two direct converters for DC-DC:



Voltage to Current



Output voltage is $+V_{in}$, 0 , or $-V_{in}$.

Switch Relations

- Output is $+V_{in}$ if 1,1 and 2,2 are on together, etc.
- A switching function representation is $v_{out}(t) = q_{11} q_{22} V_{in} - q_{12} q_{21} V_{in}$
- But KVL, KCL require $q_{11} + q_{21} = 1$, $q_{12} + q_{22} = 1$.

Switch Relations

In switching function form:

$$v_{out}(t) = q_{11}q_{22}V_{in} - q_{21}q_{12}V_{in}$$

$$i_{in}(t) = q_{11}q_{22}I_{out} - q_{21}q_{12}I_{out}$$

$$KVL+KCL: \quad q_{11} + q_{21} = 1$$

$$q_{12} + q_{22} = 1$$

$$v_{out}(t) = q_{11}q_{22}V_{in} - (1 - q_{11})(1 - q_{22})V_{in}$$



Switch Relations

$$v_{out}(t) = (q_{11} + q_{22} - 1)V_{in}$$

In this dc application, we are interested in $\langle v_{out}(t) \rangle$. The switching function averages are the duty ratios, and

$$\langle v_{out}(t) \rangle = (D_{11} + D_{22} - 1)V_{in}$$

We can choose duty ratios D_{11} and D_{22} to provide a desired $\langle v_{OUT} \rangle$.

Switch Relations

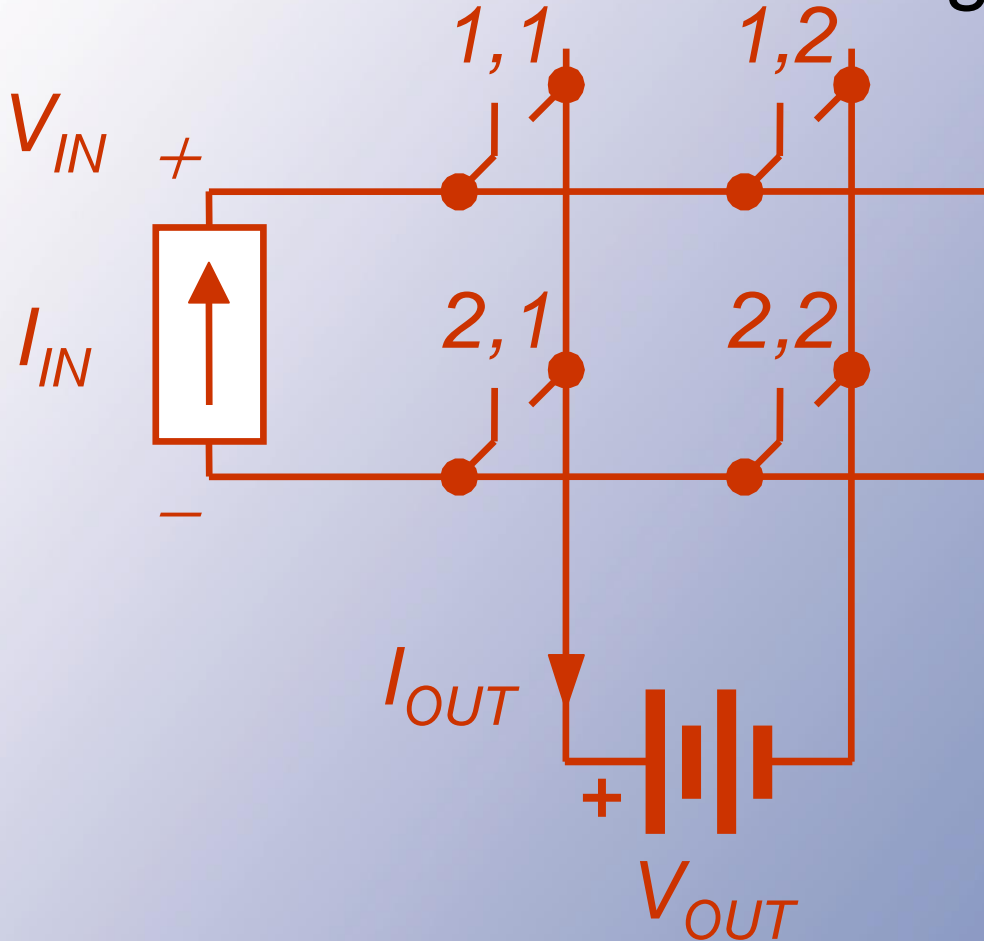
$$0 \leq D_{ii} \leq 1 \quad \Rightarrow \quad 0 \leq D_{11} + D_{22} \leq 2$$

$$\Rightarrow \quad -V_{in} \leq \langle v_{out} \rangle \leq V_{in} \quad \Rightarrow \quad \left| \langle v_{out} \rangle \right| \leq V_{in}$$

*“Buck Converter” or
“Step-Down Converter”*

$$\langle i_{in} \rangle = (D_{11} + D_{22} - 1)I_{out}$$

Current to Voltage



Output current is $+I_{in}$, 0 , or $-I_{in}$.



Switch Relations

$$\langle i_{out} \rangle = (D_{11} + D_{22} - 1) I_{in}$$

$$\langle v_{in} \rangle = (D_{11} + D_{22} - 1) V_{out}$$

$$V_{out} = \frac{\langle v_{in} \rangle}{(D_{11} + D_{22} - 1)}$$

$$0 \leq D_{ii} \leq 1 \quad \Rightarrow \quad 0 \leq D_{11} + D_{22} \leq 2$$

$$\Rightarrow \quad \left| \langle v_{out} \rangle \right| \geq V_{in}$$

Boost Converter

Summary

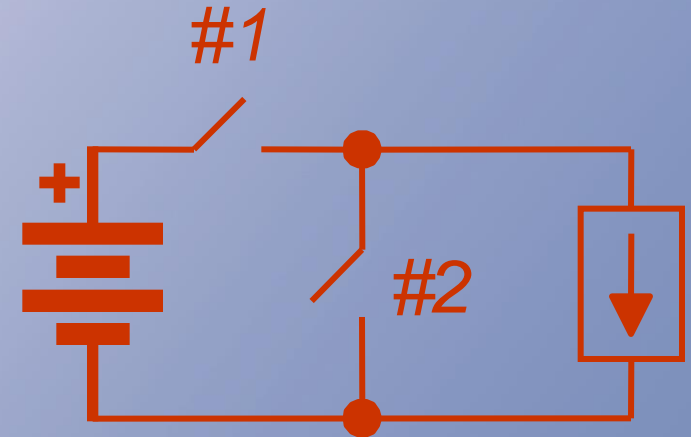
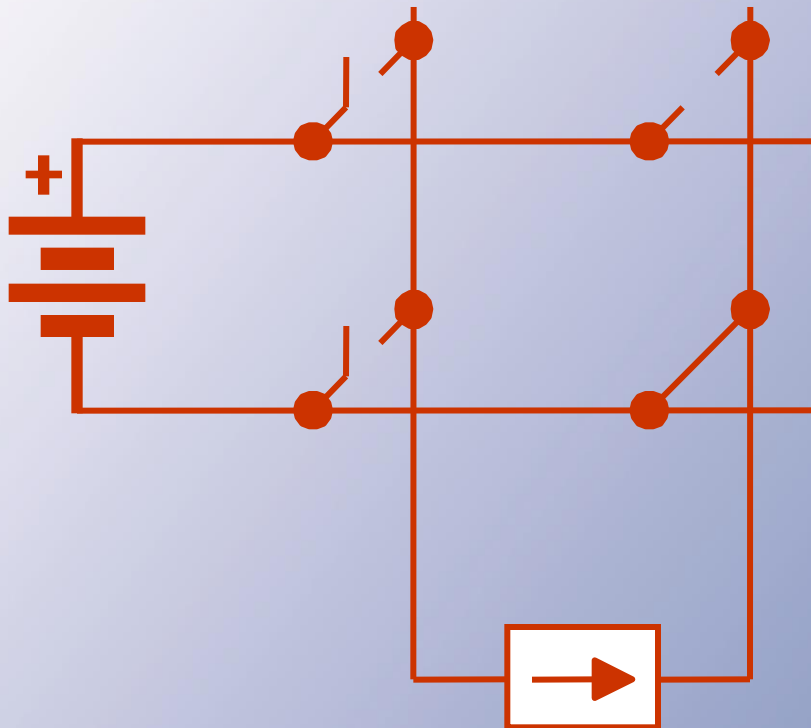
- The dc transformer is an important practical function.
- Non-switching methods, such as voltage dividers and dc regulators, are not really suitable for power conversion.
- We considered two switching circuits that accomplish buck and boost dc-dc conversion functions – types of dc transformers.

Simplifications

- In many applications, it is desirable to share a common input-output node (**ground reference**).
- This requires **one switch always on and one always off**.

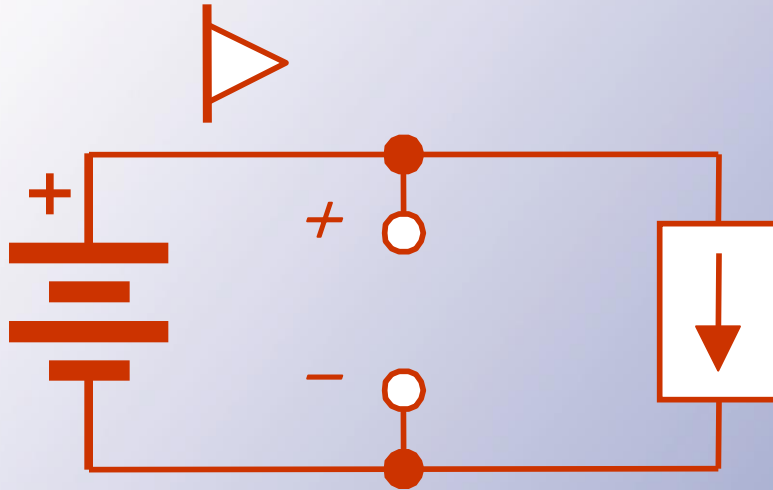
Common-Ground Dc-Dc

Example: 2x2 switch matrix, with common input-output ground

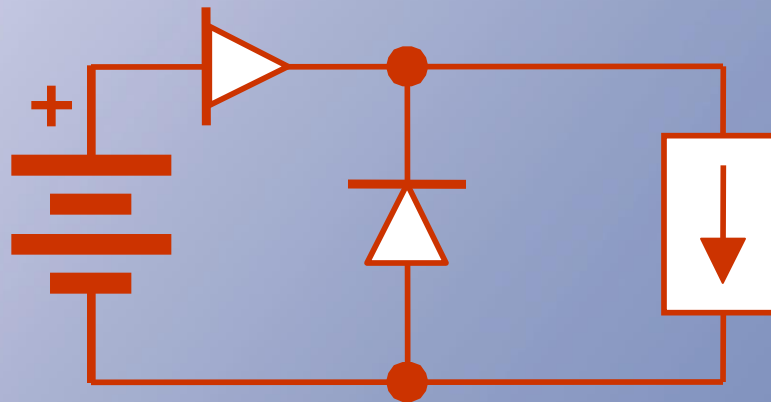
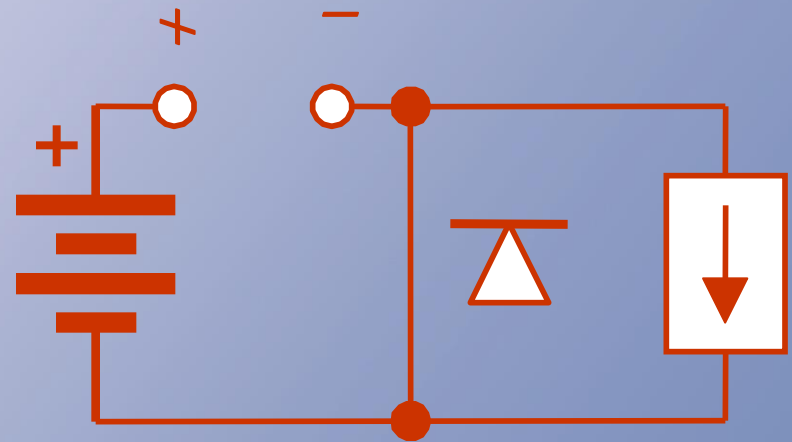


Common-Ground Dc-Dc

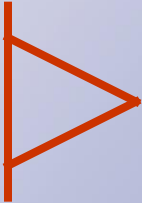
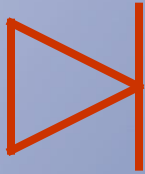
#1 ON



#2 ON



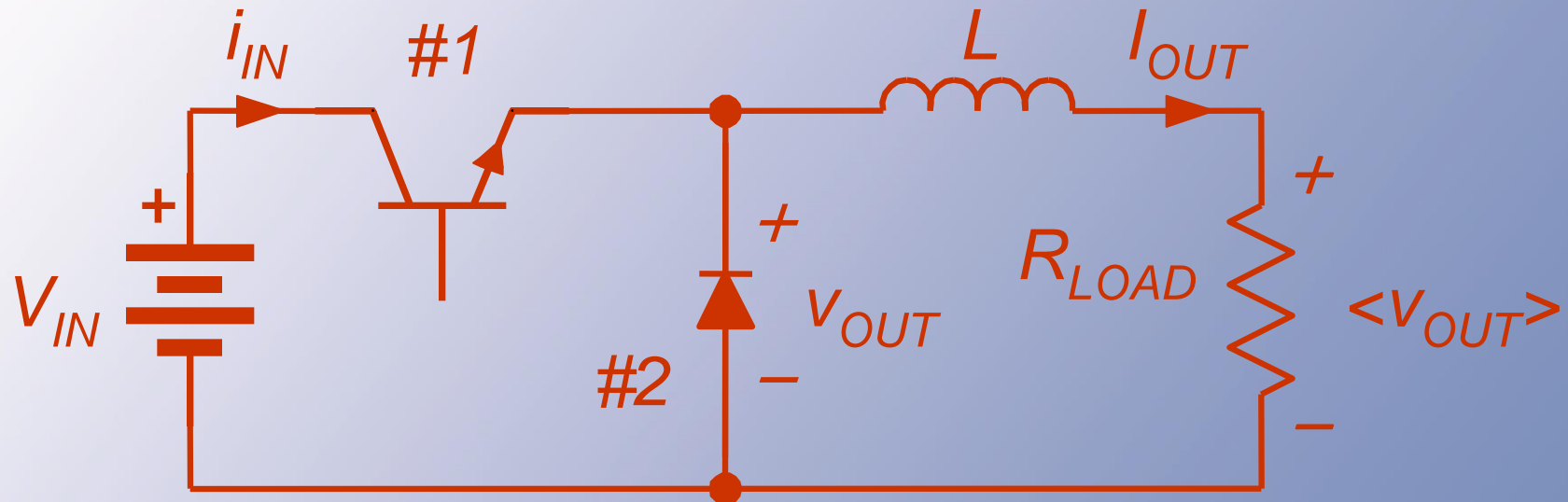
Common-Ground Dc-Dc

- With two switches left, label them **#1** and **#2**.
- One becomes  and one 
- This can be checked by testing current (on) polarity and voltage (off) polarity.

Switching Functions

- With ideal, or near-ideal, current and voltage sources, KVL and KCL require $q_1 + q_2 = 1$.
- The buck converter:

Buck Converter



- The voltage v_{out} is the “switch matrix output.”
- The load voltage is $\langle v_{out} \rangle$ since $\langle v_L \rangle = 0$.

Relationships

$$v_{out} = q_1 v_{in} \quad \langle v_{out} \rangle = D_1 v_{in}$$

$$i_{in} = q_1 i_{out} \quad \langle i_{in} \rangle = D_1 i_{out}$$

There is *no loss*.

Instantaneous power: $p_{in}(t) = q_1 v_{in} i_{out}$
 $= p_{out}(t)$

Average power: $\langle p_{out} \rangle = \langle p_{in} \rangle$
 $= D_1 v_{in} i_{out}$

Relationships

v_{out} is the switching matrix output.

$$v_{out} = q_1 V_{in} \quad \langle v_{out} \rangle = \langle q_1 V_{in} \rangle$$

$$= V_{in} \langle q_1 \rangle \rightarrow$$

load voltage

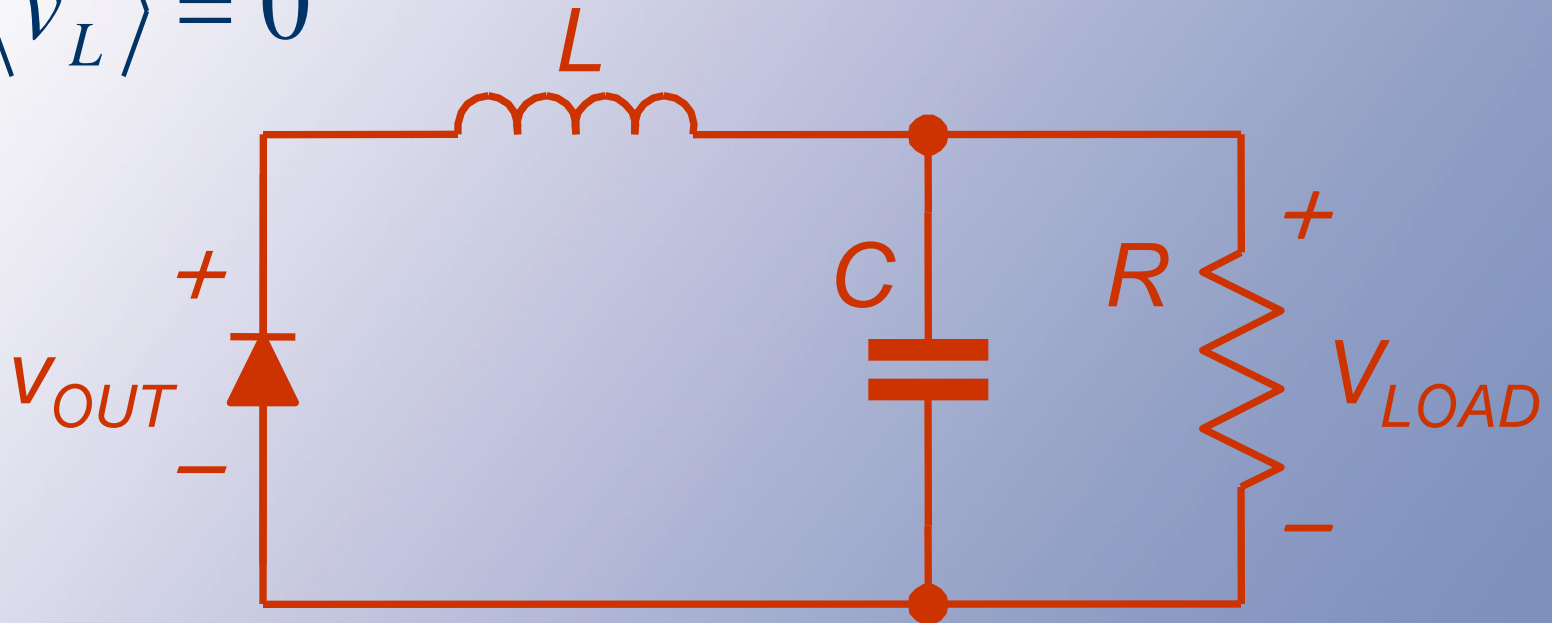
$$\rightarrow V_{out} = D_1 V_{in} \quad \langle i_{in} \rangle = \langle q_1 I_{out} \rangle$$

$$= D_1 I_{out}$$

$$\langle v_{out} \rangle = V_{out} \rightarrow \text{load voltage}$$

Relationships

$$\langle v_L \rangle = 0$$



$$V_{out} = \langle v_{out} \rangle = \langle v_{load} \rangle$$

L big $\rightarrow V_{load} \approx \text{constant}$

Relationships

$$p_{in}(t) = V_{in} i_{in}(t) \quad p_{in}(t) = p_{out}(t)$$

$$= V_{in} q_1 I_{out}$$

$$p_{out}(t) = v_{out} I_{out} \quad \langle p_{in} \rangle = \langle p_{out} \rangle$$

$$= q_1 V_{in} I_{out} \quad = D_1 V_{in} I_{out}$$

The RMS “output”

The voltage v_{out} has an RMS value of

$$\sqrt{\frac{1}{T} \int_0^T q_1(t)^2 V_{in}^2 dt} = V_{in} \sqrt{D_1}$$

Is this relevant?

Notice that $q^2(t) = q(t)$

$$q_{RMS} = \sqrt{D}$$

A Design

- A 24 V to 5 V converter, switching at 100 kHz. The nominal load is 25 W, and the ripple is to be less than 1% peak-to-peak.
- This could be met with a buck converter, since $V_{\text{out}} < V_{\text{in}}$.

A Design

- The duty ratio will need to be
 $V_{\text{out}}/V_{\text{in}} = (5 \text{ V})/(24 \text{ V}) = 0.208$
- The output current is $(25 \text{ W})/(5 \text{ V}) = 5 \text{ A}$.
- When switch #1 is on, the inductor sees
 $24 \text{ V} - 5 \text{ V} = 19 \text{ V}$.

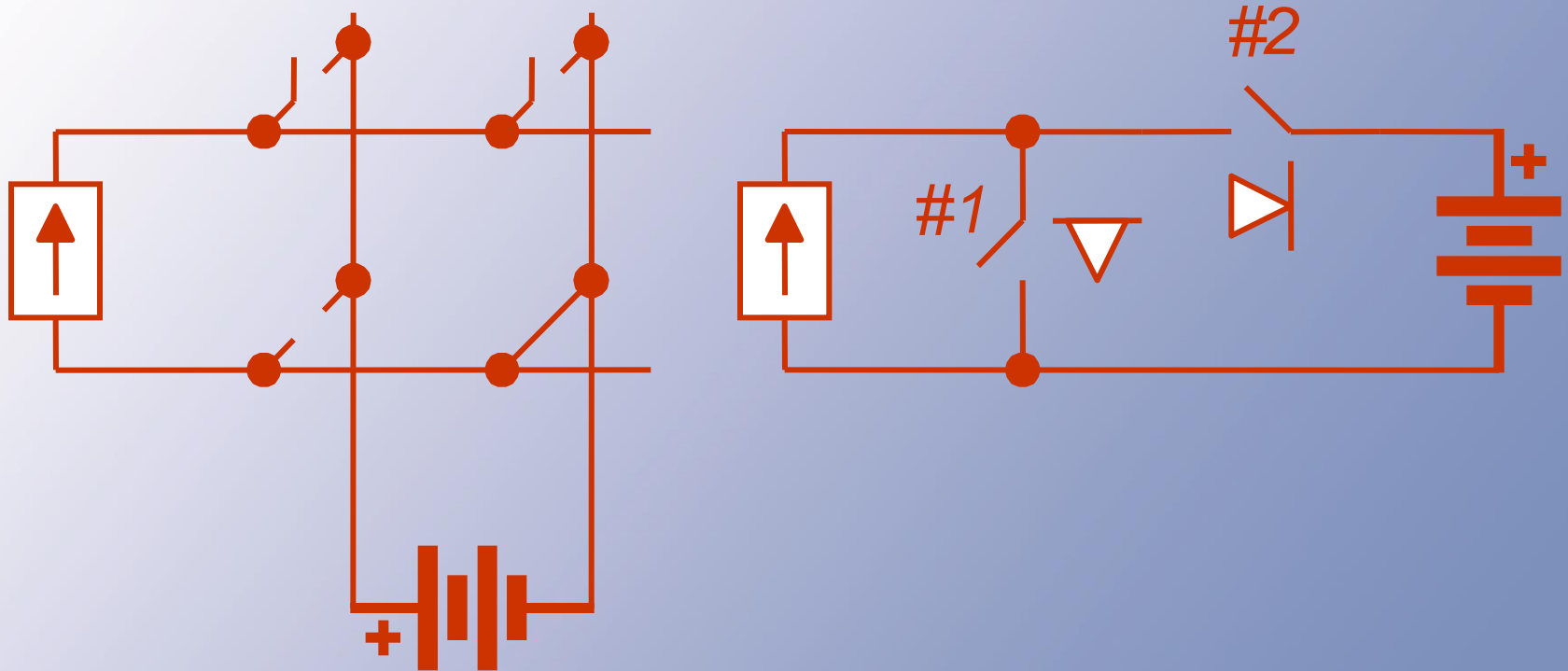
A Design

- With #1 off, the inductor sees -5V
- So, since $v_L = L di/dt$, with #1 on,
$$19 \text{ V} = L di/dt$$
$$= L \Delta i / \Delta t$$
- The time involved is 0.208 T, or 2.08 us.
We want $\Delta i < 0.01(5 \text{ A})$.
- Thus $(19 \text{ V})(2.08 \text{ us})/L < 0.05 \text{ A}$,
and $L > 0.792 \text{ mH}$

A Design

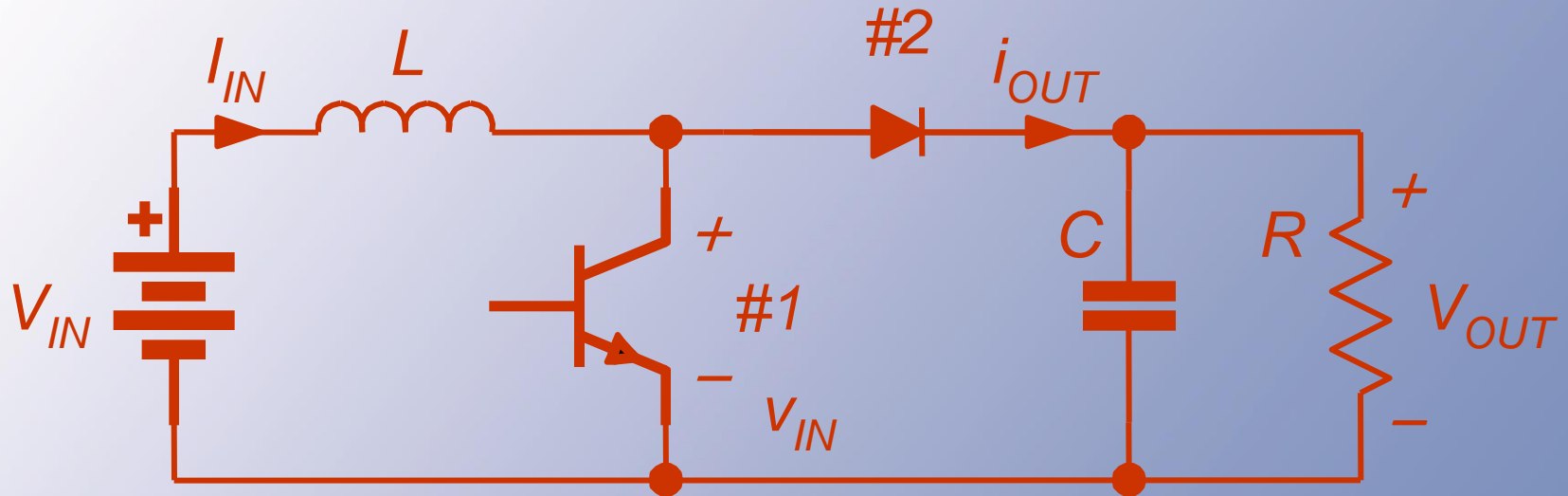
- We expect that $D_1 = 0.208$,
 $f_{\text{switch}} = 100 \text{ kHz}$, $L = 0.8 \text{ mH}$,
and $R = 1 \ \Omega$ will meet the need.
- Practice: **What is the peak-to-peak ripple if $L = 8 \ \mu\text{H}$? \rightarrow it will be 100x as big**

Boost Converter



A **boost** converter is a **buck** converter flipped horizontally.

Boost Converter



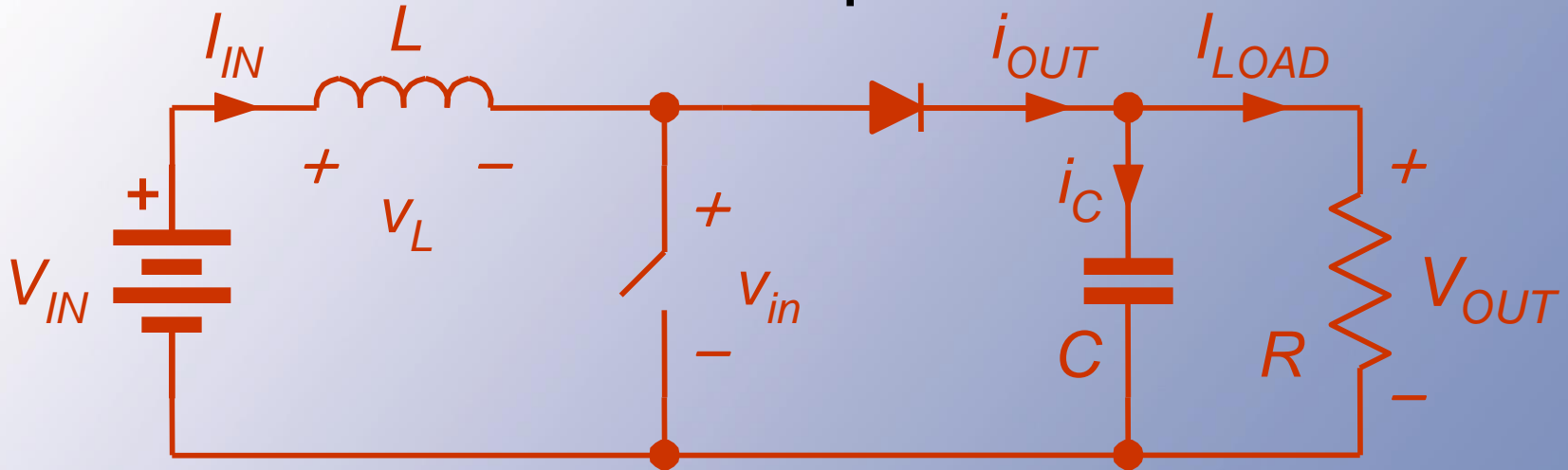
With common ground, the matrix reduces to two switches.

I_{in} is formed as a voltage in series with L .

Relationships

- The input voltage to the switch matrix is v_{in} , the voltage across the transistor.
- Since $\langle v_L \rangle = 0$, the average transistor voltage matches V_{in} .

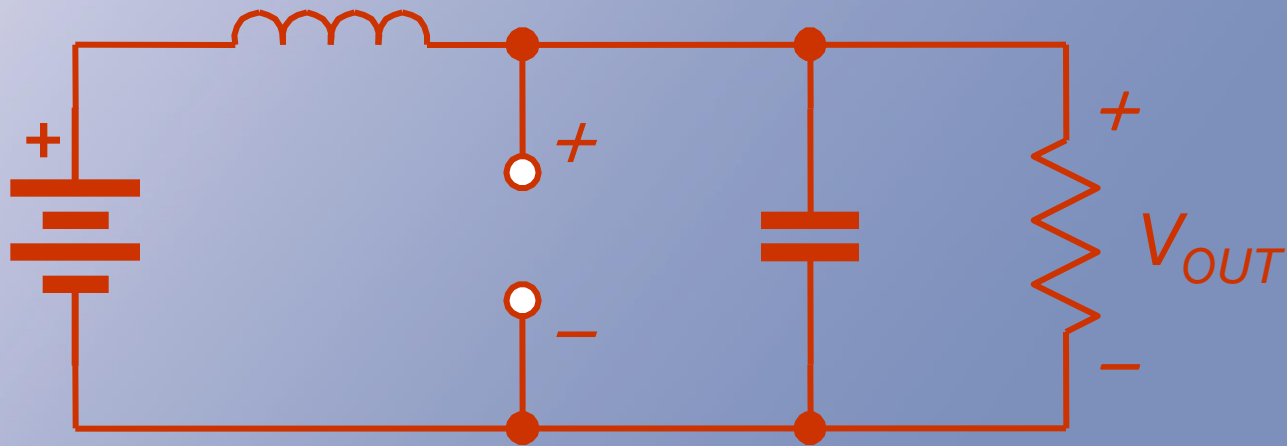
Relationships



$$\langle v_L \rangle = 0$$

$$V_{in} = \langle v_{in} \rangle$$

$$\langle i_C \rangle = 0$$



Relationships

- By KVL and KCL, **sources require**
 $q_1 + q_2 = 1.$
- Then $v_{in} = q_2 V_{out}$
 $= (1 - q_1) V_{out},$
 $i_{out} = q_2 I_{in}$
 $= (1 - q_1) I_{in}.$
- The **averages require** $\langle v_{in} \rangle = V_{in},$ and
 $V_{out} = V_{in} / (1 - D_1)$
 $= V_{in} / D_2.$

Relationships

$$i_{out} = q_2 I_{in}$$

$$v_{in} = (1 - q_1) V_{out} = (1 - q_1) I_{in}$$

$$\langle v_{in} \rangle = V_{in} = \langle (1 - q_1) \mathcal{N}_{out} \rangle$$

$$\langle i_{out} \rangle = I_{in} (1 - D_1) = I_{load}$$

$$V_{in} = V_{out} (1 - D_1) = I_{out}$$

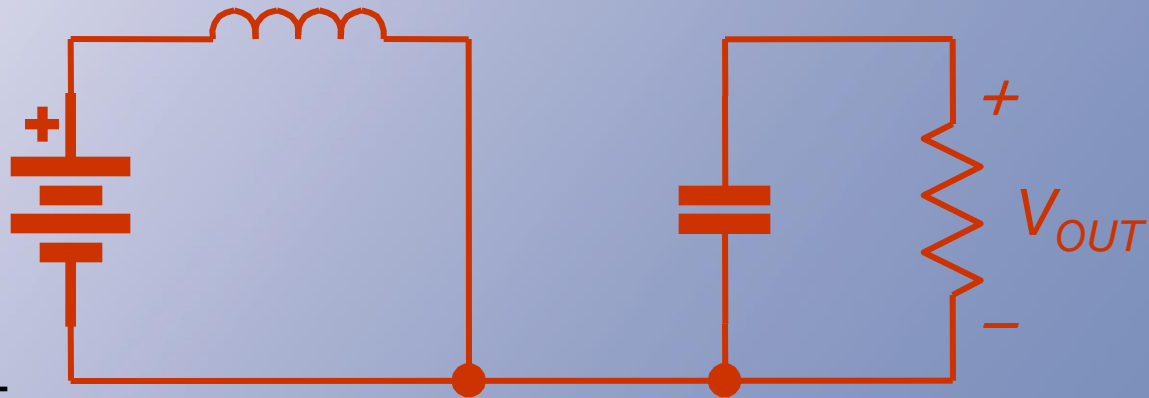
Relationships

If $D_1 = 1$:

If $D_1 \neq 1$

$$V_{out} = \frac{V_{in}}{1 - D_1}$$

$$= \frac{V_{in}}{D_2} > V_{in}$$



$V_{out} = 0$

Example

- 2 V to 5 V boost (input might be one Li-ion cell, for instance, with 2 V as its lowest value).
- Switching: 80 kHz. Load: 5 W. Input ripple: ± 10 mA. Output ripple: $\pm 1\%$.
- This gives a period of 12.5 μ s.

Boost Example

- With 2 V input and 5 V output, the load current at 5 W is 1 A, but the input current must be $(5 \text{ W})/(2 \text{ V}) = 2.5 \text{ A}$.
- With $\pm 10 \text{ mA}$ input ripple, the peak-to-peak value is 20 mA.

Boost Example

- When switch #1 is on, the inductor voltage is 2 V, and current increases.
- The duty ratios: $D_2 = V_{in}/V_{out} = 0.40$, and $D_1 = 1 - D_2 = 0.60$
- Switch #1 is on $0.60 T = 7.5 \text{ us}$.

Boost Example

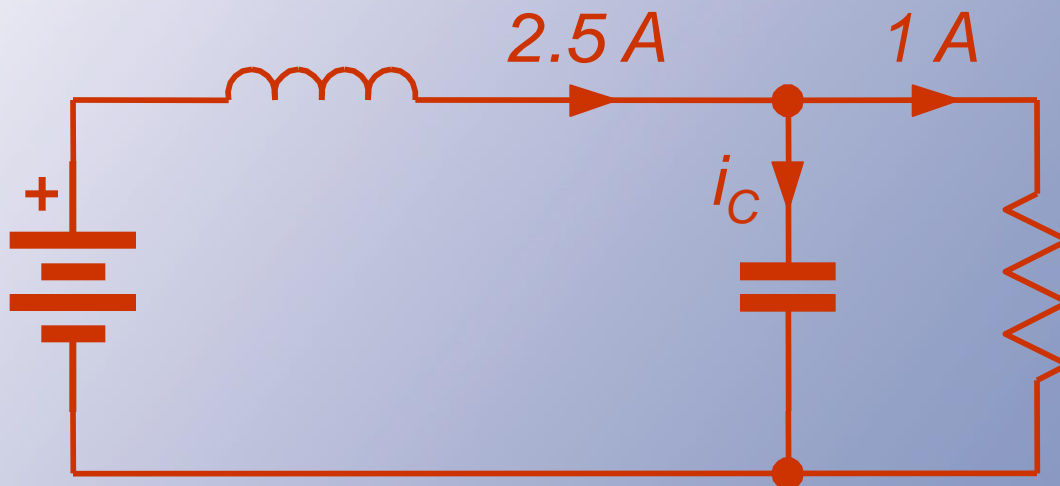
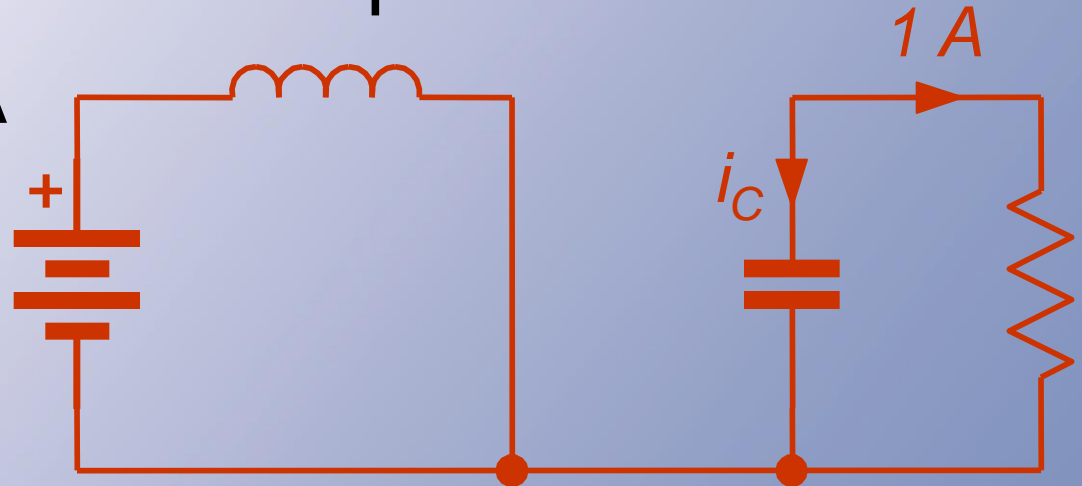
- $v_L = L \, di/dt = 2 \text{ V}$ with #1 on.
- Thus $(2 \text{ V})/L = \Delta i/\Delta t$,
 $\Delta t = 7.5 \text{ us}$.
- To get $\Delta i < 0.02 \text{ A}$, we need
 $L > (2 \text{ V})(7.5 \text{ us})/(0.02 \text{ A})$, or
 $L > 0.75 \text{ mH}$.

Boost Example

- What about V_{out} ?
- The capacitor current is
 $I_{\text{in}} - I_{\text{load}} = 2.5 \text{ A} - 1 \text{ A}$
when switch #2 is on, and
 -1 A when switch #1 is on.
- We want $\pm 1\%$ of 5 V, or a peak-to-peak change below 0.1 V.

Boost Example

#1 ON: $i_C = -1 \text{ A}$



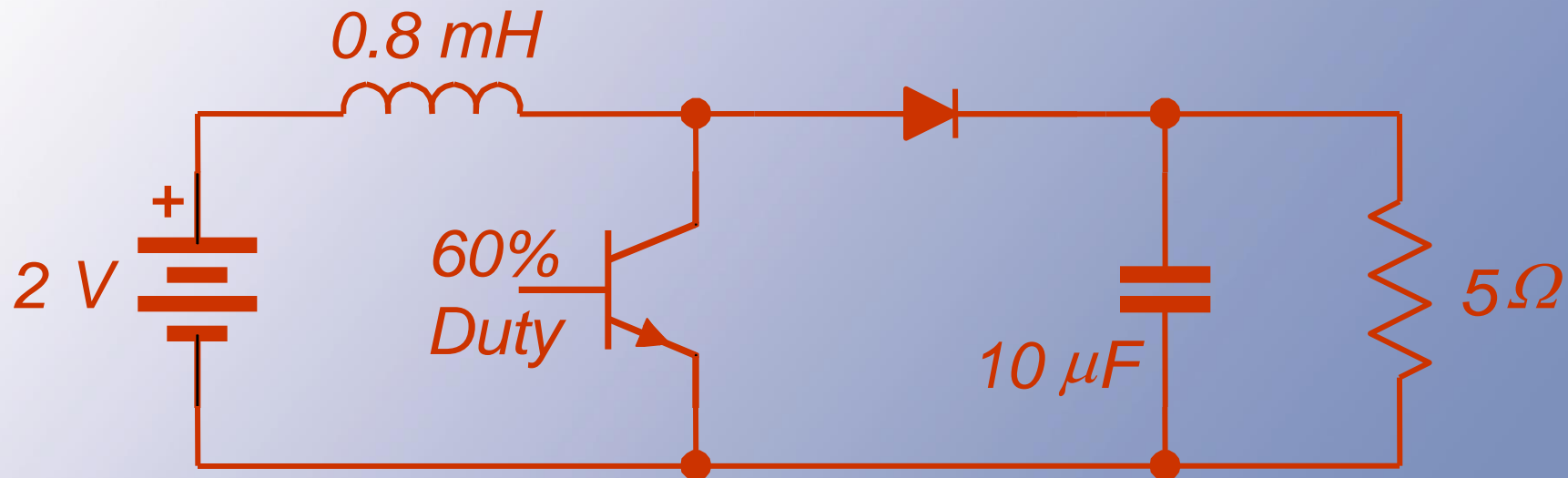
#2 ON: $i_C = 1.5 \text{ A}$

Boost Example

- With switch #2 on (duty ratio was found to be 0.4, so time is 5 us),
$$i_C = 1.5 \text{ A}$$
$$= C \, dv/dt$$
$$= C \, \Delta v/\Delta t.$$
- $(1.5 \text{ A})(5 \text{ us})/C = \Delta v < 0.1 \text{ V}.$
- This requires $C > 75 \text{ uF}.$

Boost Example

2 to 5 V, 80 kHz boost converter:

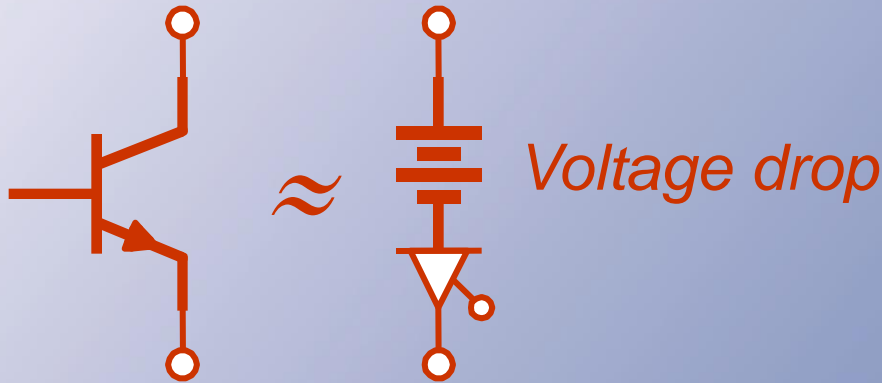
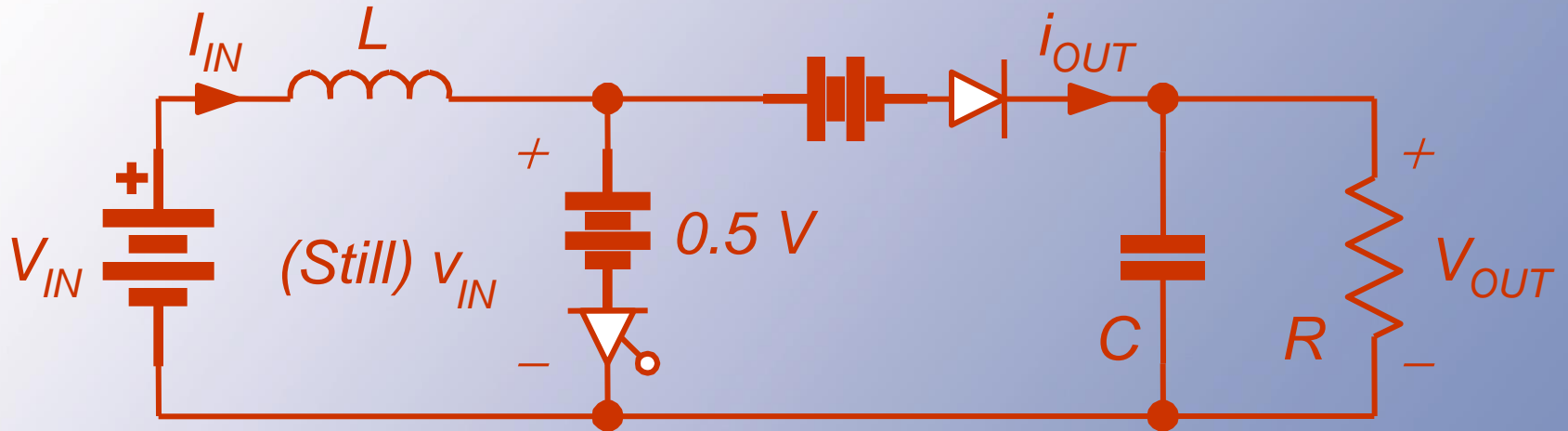


Practice: What if f_s is changed to 40 kHz? → average values are the same, ripple 2x

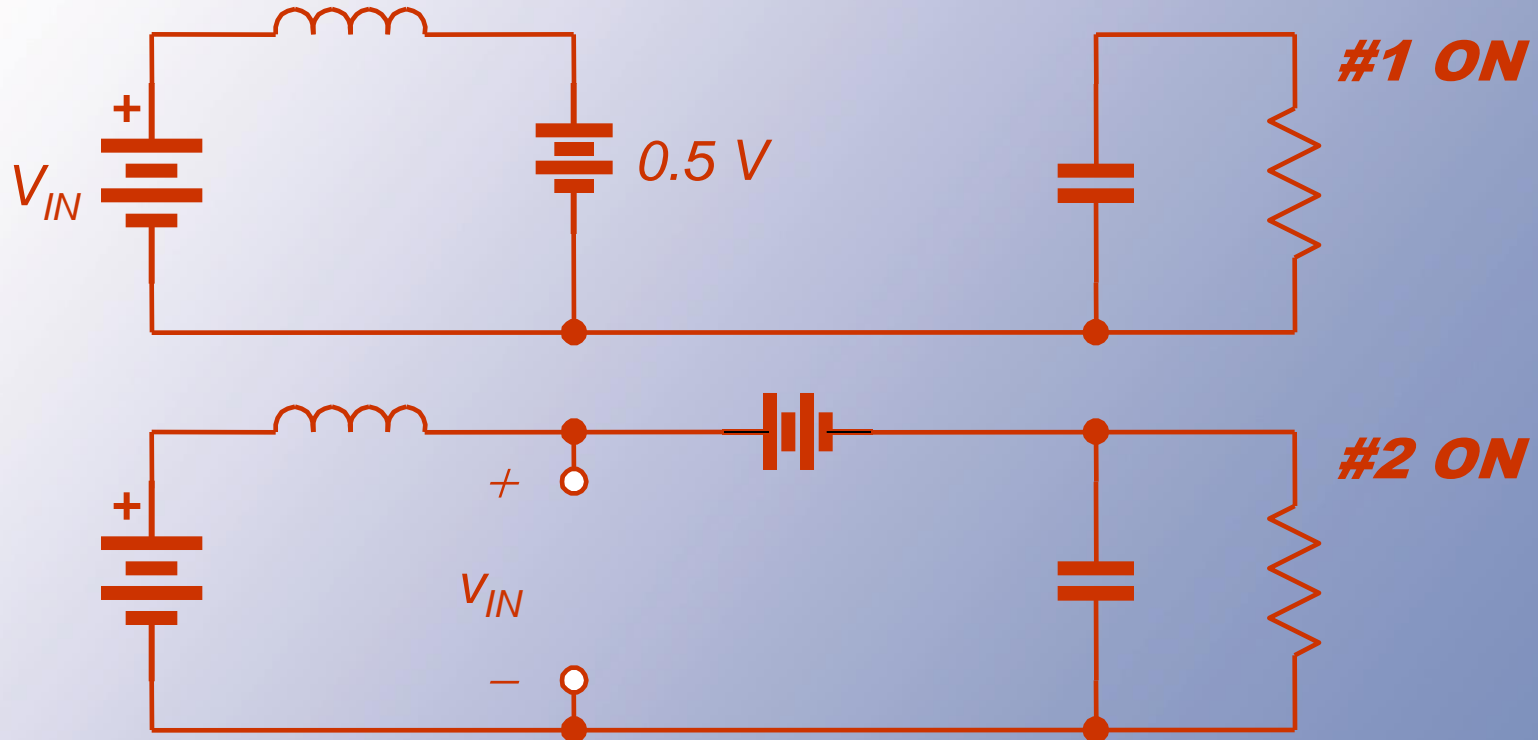
Comments

- With a few practice examples, **you should be able to design a common-ground buck or boost converter.**
- Challenge: Think about effects of nonideal switching.
- It is not so difficult to include some basic nonideal effects, such as switching device voltage drops and resistances.
- Consider an example with switch and diode voltage drop.

Nonideal boost



Nonideal boost



$$v_{in} = q_1(0.5 V) + q_2(V_{out} + 1)$$

$$V_{in} = \langle v_{in} \rangle = D_1(0.5 V) + (1 - D_1)(V_{out} + 1)$$

Nonideal boost

- Switching function expressions still apply.
- Boost: $v_{in} = q_1(0.5 V) + q_2(V_{out} + 1 V)$.

- On average,

$$\begin{aligned}\langle v_{in} \rangle &= V_{in} \\ &= D_1(0.5V) + (1-D_1)(V_{out} + 1 V), \text{ and} \\ V_{out} &= (V_{in} + 0.5D_1 - 1)/(1 - D_1)\end{aligned}$$

- For current, $i_{out} = q_2 I_L$, $\langle i_{out} \rangle = D_2 I_L$.
- Since $\langle i_{out} \rangle$ is the load current I_{load} , we have $I_L = I_{load}/D_2 = I_{load}/(1 - D_1)$.

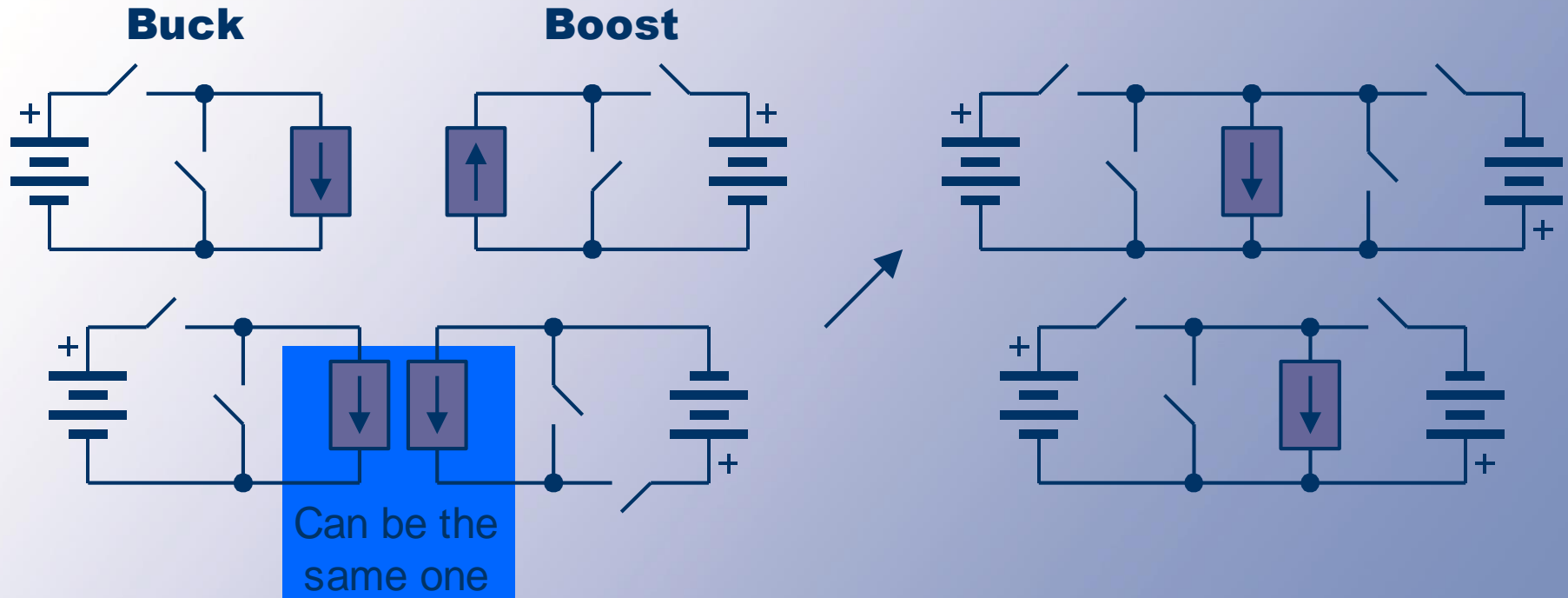
Nonideal boost

- The efficiency: $P_{in} = V_{in} I_L$, $P_{out} = V_{out} I_{load}$.
- So $P_{in} = V_{in} I_{load} / (1 - D_1)$ and
$$P_{out} = (V_{in} + 0.5D_1 - 1) I_{load} / (1 - D_1)$$
- The efficiency ratio $\eta = (V_{in} + D_1/2 - 1) / V_{in}$,
and $\eta = 1 - (1 - D_1/2) / V_{in}$.
- This is less than 100%, reflecting the losses in the switch forward drops.
- Switching functions support analysis of converters even with these extra parts.

Indirect Dc-Dc Converters

- The buck is a dc transformer with $V_{out} < V_{in}$.
- The boost gives $V_{out} > V_{in}$.
- How can we give full range? *Use a buck as the input for a boost.*
- That is, use the current source output of a buck to provide the input source for a boost.
- Remove redundant or unnecessary switches. Result is the polarity reverser: buck-boost.

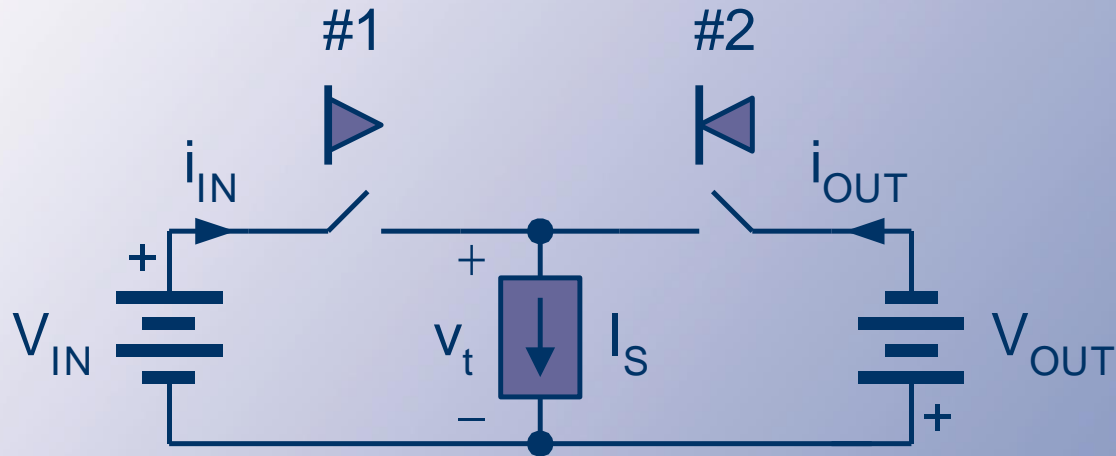
Buck-Boost Development



Final Simplification

- The switch across the current source is not necessary for KCL.
- Try removing it.
- The current source is a *transfer source*.

Buck-Boost Converter



Buck-Boost



Left switch is FCFB. Right switch is FCRB.

Relationships

- To meet KVL and KCL, $q_1 + q_2 = 1$.
- There are really two matrices now. Let us consider the transfer source, which is manipulated by both matrices.
- Transfer voltage is subject to control.
- Transfer voltage $v_t = q_1 V_{in} - q_2 V_{out}$.
- Transfer source power is $v_t I_s = q_1 V_{in} I_s - q_2 V_{out} I_s$.
- We want the average power in the transfer source to be zero -- no loss.

Relationships

KVL + KCL:

$$q_1 + q_2 = 1$$

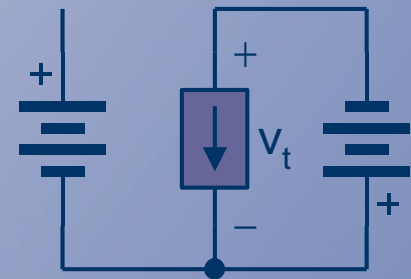
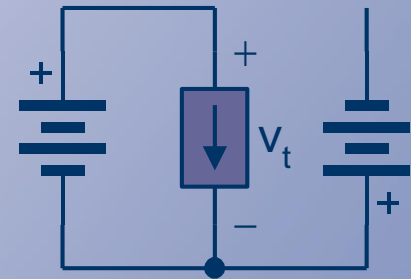
$$v_t = q_1 V_{in} - q_2 V_{out}$$

$$v_t I_s = q_1 V_{in} I_s - q_2 V_{out} I_s$$

$$\langle v_t \rangle = D_1 V_{in} - D_2 V_{out}$$

$$\langle v_t I_s \rangle = I_s \langle v_t \rangle = I_s (D_1 V_{in} - D_2 V_{out})$$

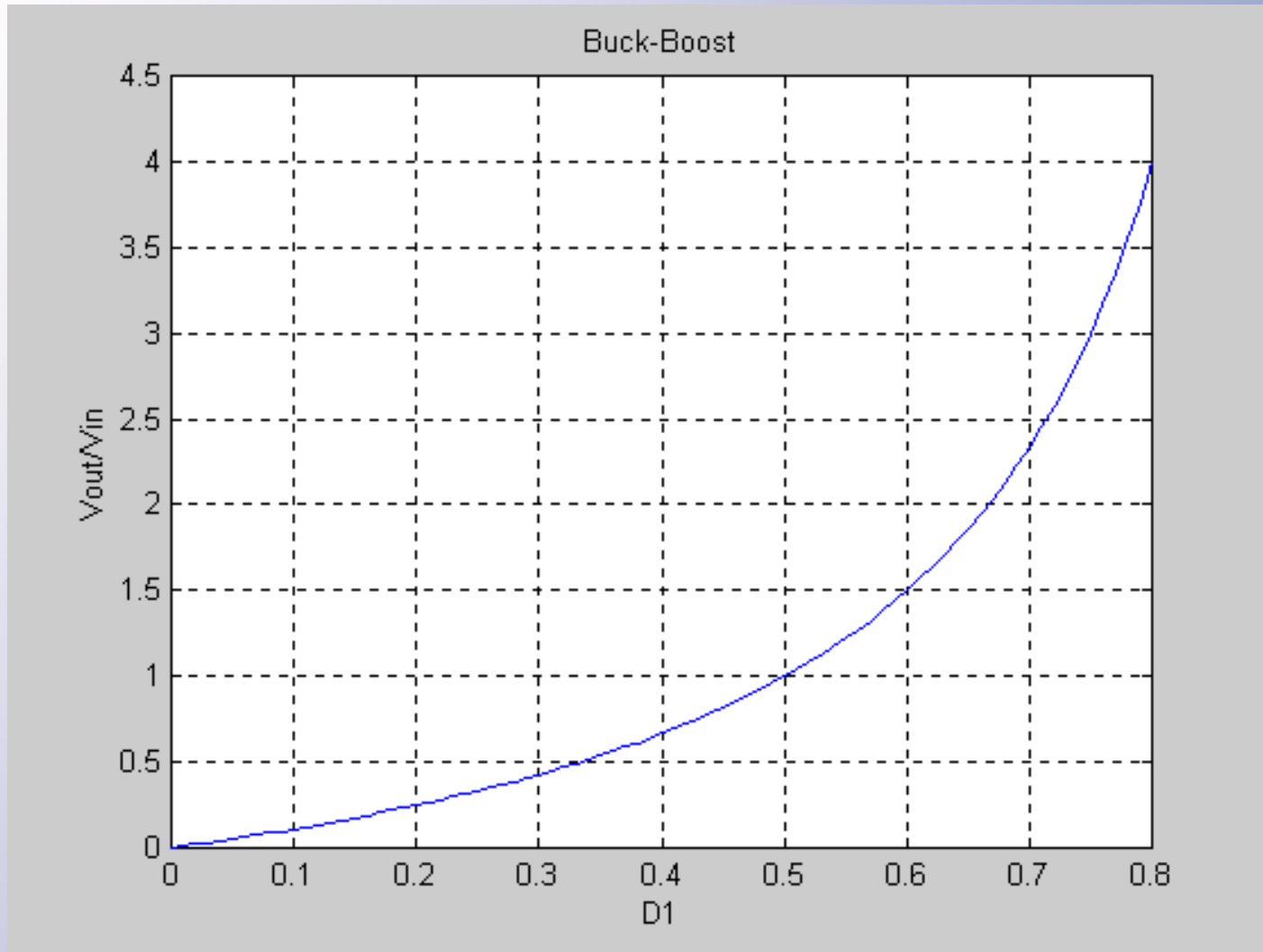
$\langle v_t I_s \rangle$ must be zero, not to have losses in the transfer source.



Relationships

- This can be done if $D_1 V_{in} = D_2 V_{out}$.
- Since $D_1 + D_2 = 1$, we have $D_1 V_{in} = (1 - D_1) V_{out}$.
- This becomes $V_{out} = D_1 V_{in} / (1 - D_1)$.
- The polarity reversal comes from the cascade process.

Buck-Boost



Relationships

- The buck-boost allows outputs both higher and lower than the input, but a polarity shift is present.
- The transfer source can be an inductor alone to avoid loss.

Relationships



Consumes no average power.
Maintains fixed I .

Can be approximated by an inductor.



This will be our transfer
current source.

What About Currents?

- The input current: $i_{in} = q_1 I_s$,
- The output current: $i_{out} = q_2 I_s$,
- Average input: $I_{in} = D_1 I_s$,
- Average output: $I_{out} = D_2 I_s$.
- We do not really know I_s . Add the above:
 $I_{in} + I_{out} = (D_1 + D_2)I_s = I_s$.

Currents and Stresses

- The transfer source sees a current equal to the sum of input and output average currents.
- Each switch must carry I_s , and each must block $V_{in} + V_{out}$.
- All device ratings are higher than either the input or output needs.