Power Electronics

Fourier Series and Their Applications to Power Electronics; Distortion and Regulation

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Why Fourier?

- Switch action is periodic by design.
- We often have specific input frequencies, and seek specific output frequencies, but many frequencies occur together.
- These mean that we need to explore frequency content of our waveforms.

The Basics

Any physically realizable periodic function, *f(t)* = *f(t+T)*, (period *T*) can be written as a sum of sinusoids:

$$f\left(t\right) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega t) + b_n \sin(n\omega t)$$

where the sum is taken over *n*=1 to infinity, $\omega = 2\pi/T$, and the a_n and b_n coefficients are given by explicit integral equations,

The Basics

$$f(t) = f(t + T)$$

$$f(t) = a_0 + \sum_{n=1}^{\infty}\left[a_n \cos(n\omega t) + b_n \sin(n\omega t)\right]$$

$$\omega = \frac{2\pi}{T}, \quad freq = \frac{1}{T}, \quad \omega = 2\pi freq$$

a_0 : Average of $f(t) = \langle f(t) \rangle$

The series works, provided the coefficients are:

$$a_0 = \frac{1}{T}\int_{\tau}^{\tau+T} f(t)dt$$

$$a_n = \frac{2}{T}\int_{\tau}^{\tau+T} f(t)\cos(n\omega t)dt$$

$$b_n = \frac{2}{T}\int_{\tau}^{\tau+T} f(t)\sin(n\omega t)dt$$

Notice that the integrals are taken over a period – but it is fine to start anywhere.

Another Form

We can also write

$$f(t) = \sum_{n=0}^{\infty} c_n \cos(n\omega t + \theta_n)$$

This form is common
in electrical engineering and works if:

Another Form

$$f(t) = \sum_{n=0}^{\infty} c_n \cos(n\omega t + \theta_n)$$

$$c_0 = a_0, \quad \theta_0 = 0$$

$$c_n = \sqrt{a_n^2 + b_n^2}$$

$$\theta_n = -\tan^{-1}\left(\frac{b_n}{a_n}\right)$$

Some Terminology

- Each cosine term, $c_n \cos(n\omega t + \theta_n)$, is called a *Fourier component* or a *harmonic* of the function *f(t).* We call each the *n*th harmonic.
- The value c_n is the component amplitude; θ_n is the component phase.

Some Terminology

- $c_0 = a_0$ is the dc component, equal to the average value of *f(t)*, $c_0 = \langle f(t) \rangle$.
- The term $c_1 \cos(\omega t + \theta_1)$ is the *fundamental* of *f(t)*, while $1/T$ is the *fundamental frequency*.

Some Terminology

- In most converters, we seek a single desired frequency (perhaps the output frequency). This is associated with a single *wanted component*.
- All others are *unwanted components*.

Angular Time

- The change of variables $\theta = \omega t$ is often useful. In many cases, the waveform shape, rather than explicit timing, is the important issue.
- The variable θ is *angular time*.
- This is strictly a change of variables.

Angular Time

$$a_n = \frac{2}{T}\int_{\tau}^{\tau+T} f(t)\cos(n\omega t)\,dt$$

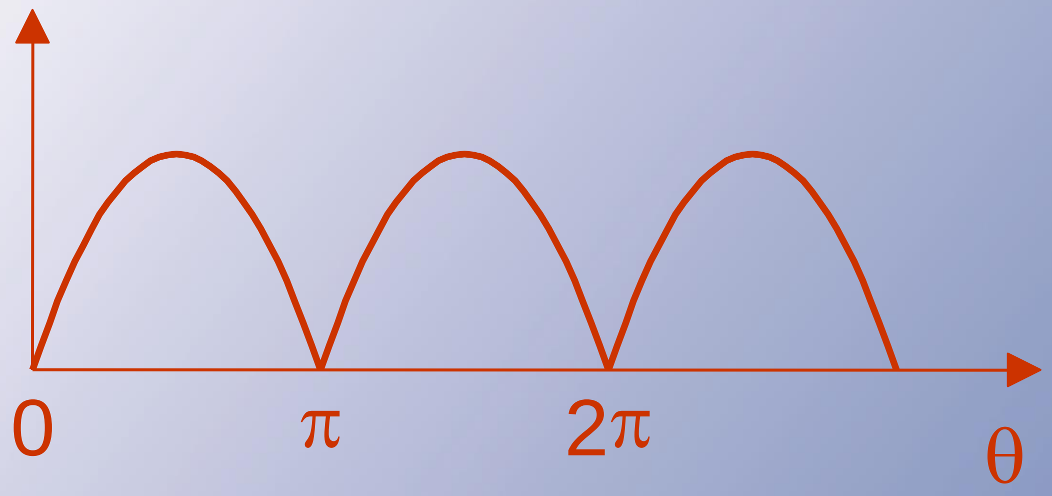
$$\theta = \omega t, \qquad \omega = \frac{2\pi}{T}$$

$$\omega T = 2\pi = \theta\frac{T}{t}$$

$$a_n = \frac{2}{2\pi}\int_{\theta_0}^{\theta_0+2\pi} f(\theta)\cos(n\theta)\,d\theta$$

Angular Time

θ : Angular Time

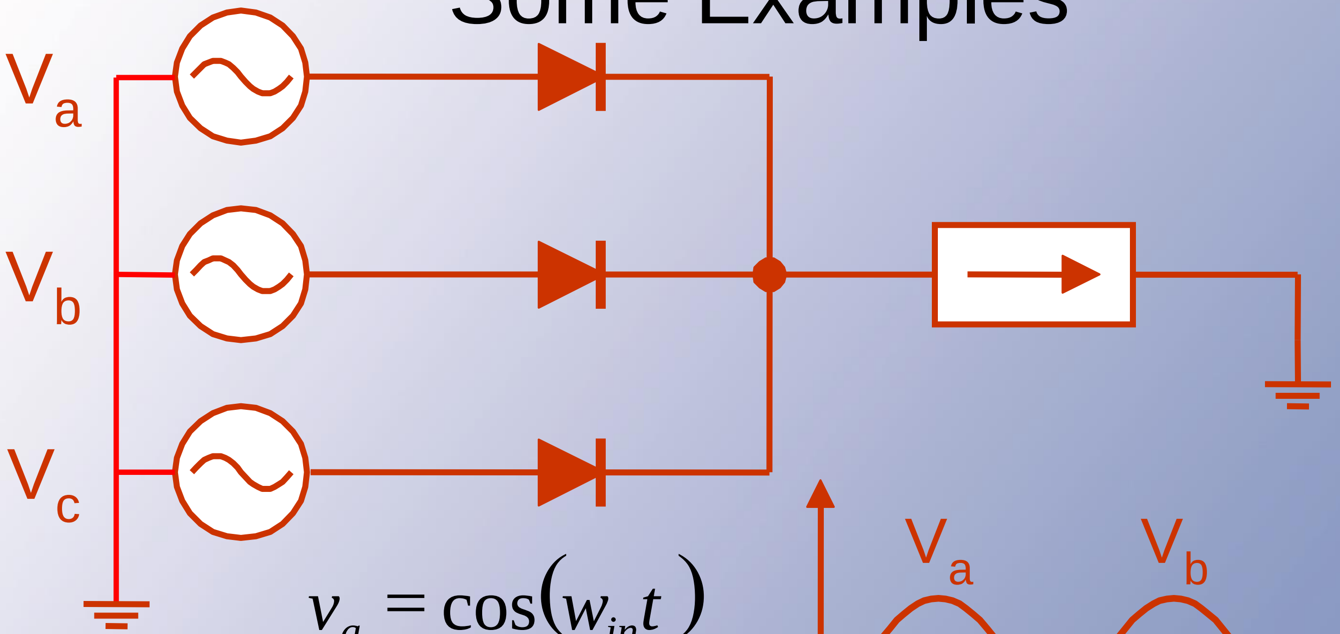


Useful when the shape of the waveform is important. Frequency will not matter.

Why Fourier Analysis?

- Many converters create waveforms by “chopping up” pieces of sinusoids.
- Fourier analysis applies readily to piecewise sinusoidal waveforms.
- Identifies the dc and various ac frequency components created.
- Establishes conditions on whether a conversion is successful.

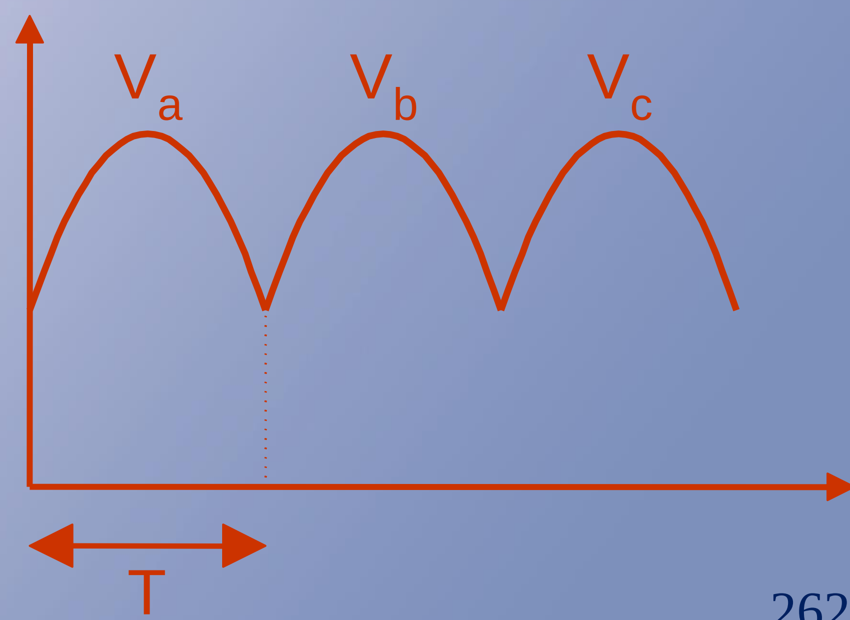
Some Examples



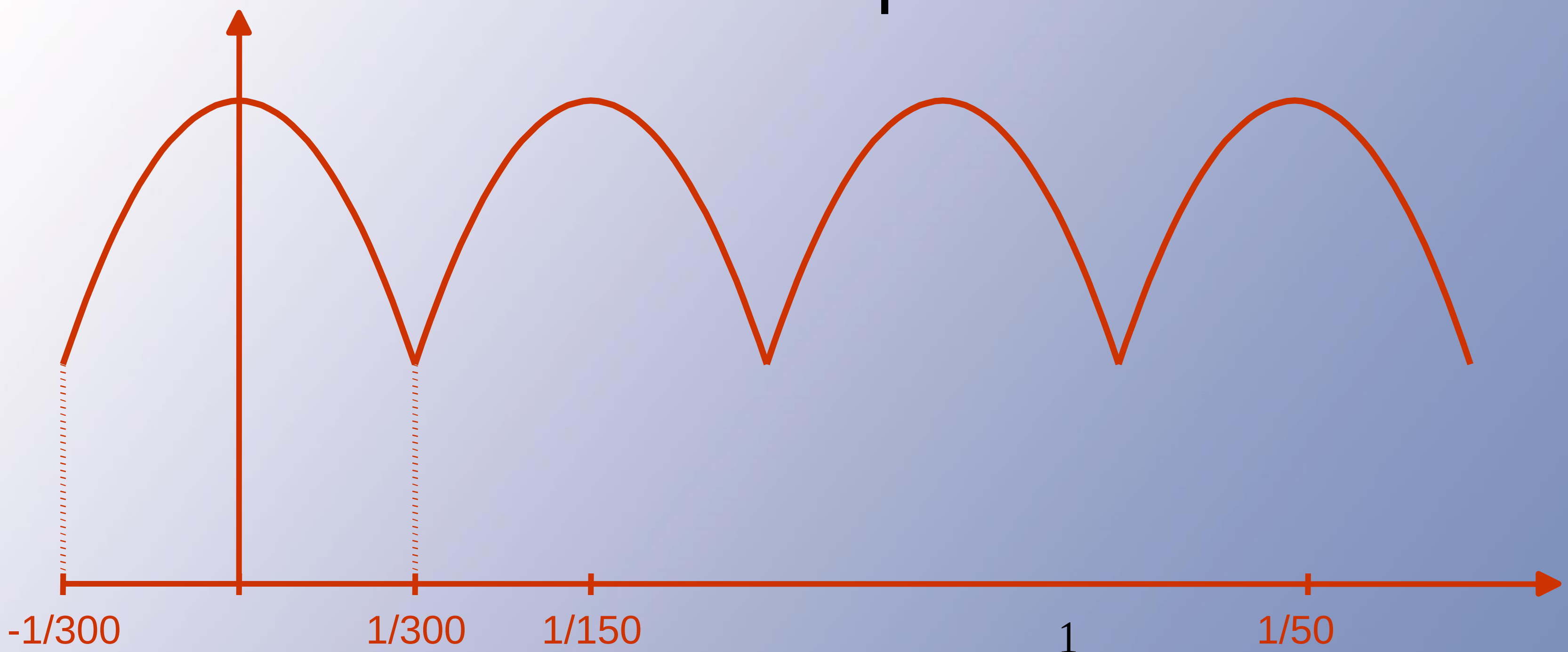
$$v_a = \cos(w_{in} t)$$

$$w_{in} = 100\pi \frac{rad}{s}$$

$$w = \frac{2\pi}{T} \neq w_{in}$$



Some Examples



Period $T = \frac{1}{150}\, s$

$$a_0 = 150 \int_{-\frac{1}{300}}^{\frac{1}{300}} V_0 \cos(100\pi t)\, dt$$

Some Examples

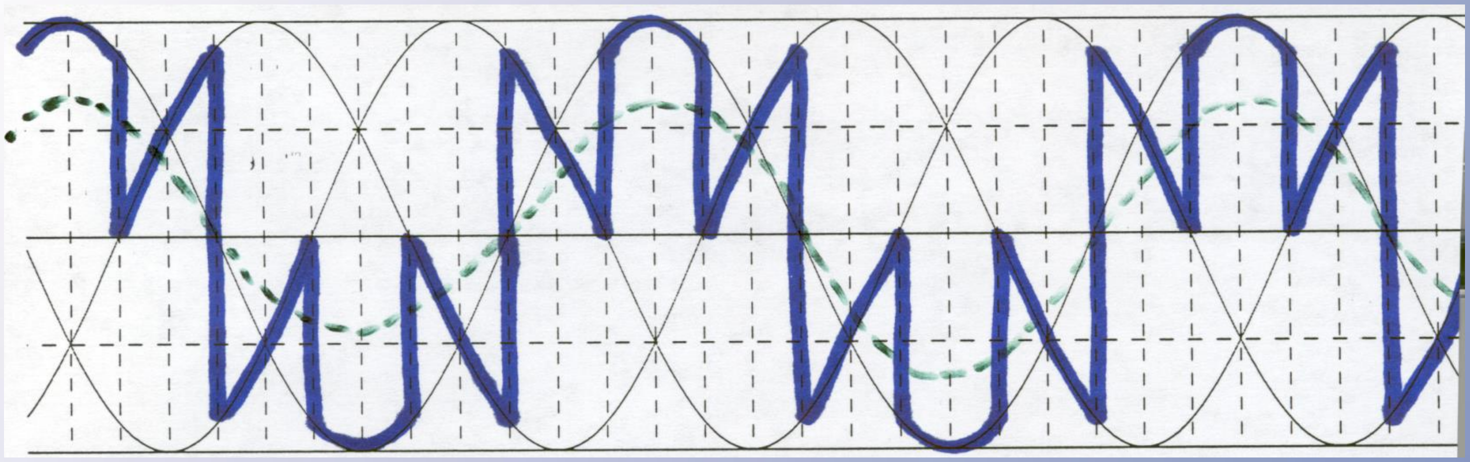
$$a_n = 2 \cdot 150 \int_{-\frac{1}{300}}^{\frac{1}{300}} V_0 \cos(100\pi t)\cos(n300\pi t)\,dt$$

$$\omega = \frac{2\pi}{T} = 300\pi$$

a_1 : 150 Hz component

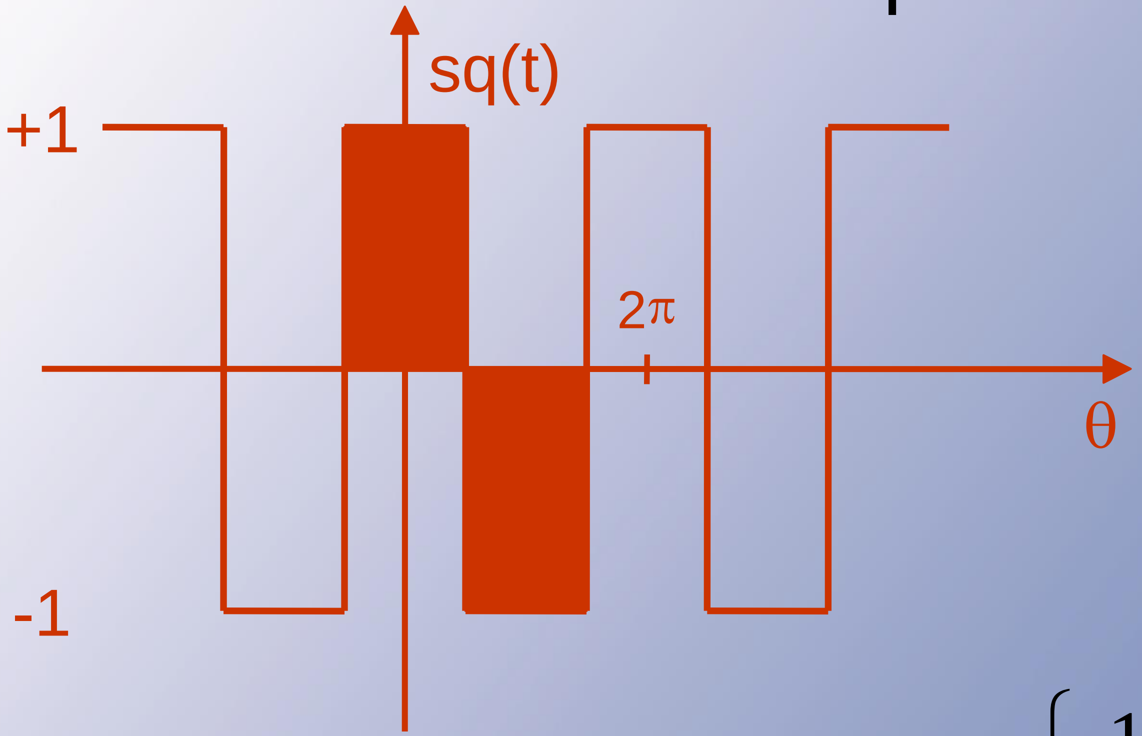
Some Examples

Piecewise sinusoidal:



$$a_0 = \frac{1}{2\pi}\left[\int_{\frac{-\pi}{6}}^{\frac{\pi}{6}} v_a(\theta)d\theta + \int_{\frac{\pi}{6}}^{\frac{3\pi}{6}} v_b(\theta)d\theta + \int_{\frac{3\pi}{6}}^{\frac{5\pi}{6}} v_c(\theta)d\theta + \int_{\frac{5\pi}{6}}^{\frac{7\pi}{6}} v_a(\theta)d\theta + \ldots\right]$$

Another Example



$$sq(t) = \mathrm{sgn}[\cos(t)] \qquad \mathrm{sgn}(x) = \begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases}$$

Another Example

Analysis of the square wave:

$$a_0 = 0$$

$$a_n = \frac{1}{\pi}\left[\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 1\cos(n\theta)\,d\theta + \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} -1\cos(n\theta)\,d\theta\right]$$

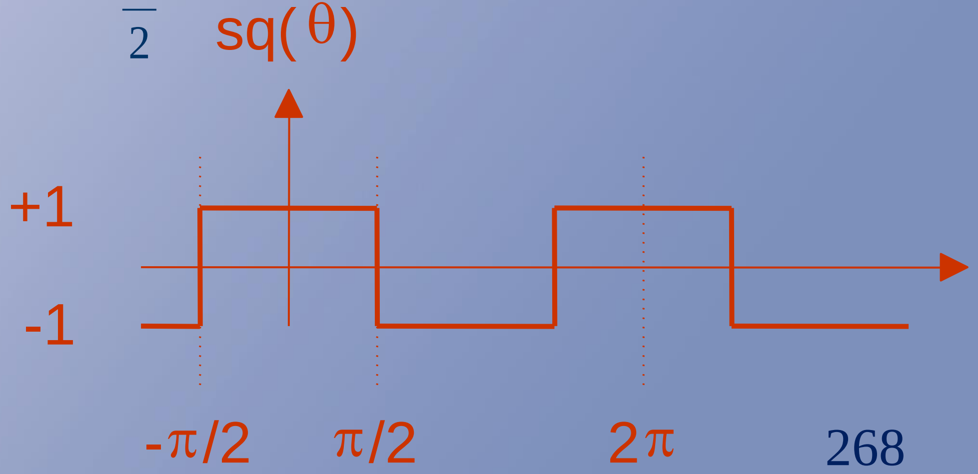
The Terms

$$a_n = \frac{1}{\pi}\int_{-\frac{\pi}{2}}^{\frac{3\pi}{2}} V_0 sq(\theta)\cos(n\theta)d\theta$$

$$= \frac{1}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} V_0 \cos(n\theta)d\theta - \frac{1}{\pi}\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} V_0 \cos(n\theta)d\theta$$

$$= \frac{4V_0}{n\pi}\sin\left(\frac{n\pi}{2}\right)$$

$$a_0 = 0$$



The Terms

$$a_n = \frac{1}{\pi}\left[\frac{\sin(n\theta)}{n}\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} - \frac{1}{\pi}\left[\frac{\sin(n\theta)}{n}\right]_{\frac{\pi}{2}}^{\frac{3\pi}{2}}$$

$$= \frac{1}{n\pi}\left[\sin\left(\frac{n\pi}{2}\right) - \sin\left(\frac{-n\pi}{2}\right) - \sin\left(\frac{3n\pi}{2}\right) + \sin\left(\frac{n\pi}{2}\right)\right]$$

$$\sin(x) = -\sin(-x)$$

$$\sin\left(\frac{3n\pi}{2}\right) = \sin\left(\frac{n\pi}{2} + \frac{2n\pi}{2}\right)$$

$$= \sin\left(\frac{n\pi}{2} + n\pi\right)$$

The Terms

$$a_n = \frac{1}{n\pi}\left[\sin\left(\frac{n\pi}{2}\right)(4)\right]$$

$$a_n = \frac{4}{n\pi}\sin\left(\frac{n\pi}{2}\right)$$

$$(= 0 \text{ for n even})$$

The Terms

$$b_n = \frac{1}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} +1\sin(n\theta)\,d\theta + \frac{1}{\pi}\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} (-1)\sin(n\theta)\,d\theta$$

$$= \frac{1}{\pi}\left[\frac{-\cos(n\theta)}{n}\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} + \frac{-1}{\pi}\left[\frac{-\cos(n\theta)}{n}\right]_{-\frac{\pi}{2}}^{\frac{3\pi}{2}}$$

$$= \frac{-1}{n\pi}\left[\cos\left(\frac{n\pi}{2}\right) - \cos\left(\frac{-n\pi}{2}\right)\right]$$

$$\cos(x) = \cos(-x)$$

$$b_n = 0$$

The Terms

So: $a_n = \frac{4}{n\pi}\sin\left(\frac{n\pi}{2}\right)$ $b_n = 0$

sq(θ): Fourier components at n = 1,3,5, …

Component amplitudes as $\frac{1}{n}$.

Fundamental: $n = 1$

$$\frac{4}{\pi}\sin\left(\frac{\pi}{2}\right)\cos(\theta) \qquad f(t) = \frac{4}{\pi}\cos(wt)$$

Given: Square wave is in phase with cosine

The Square Wave

- Series is

$$sq(\theta) = \sum_{n=1}^{\infty} \frac{4}{n\pi} \sin\left(\frac{n\pi}{2}\right) \cos(n\theta)$$

- More examples can be found in Appendix C.

The Terms

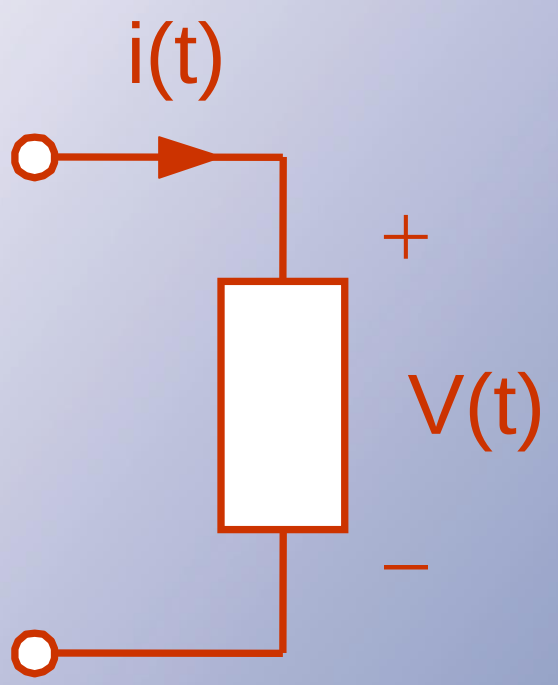
If the square wave is in phase with the sine, it can be represented as sgn [sin(*wt*)].

$$a_n = 0 \qquad b_n = \frac{4}{n\pi}\sin\left(\frac{n\pi}{2}\right)$$

$$f(t) = \sum_{n=0}^{\infty} c_n \cos(nwt + \theta_n)$$

$$sq(t) = \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{\sin\left(\frac{n\pi}{2}\right)}{n}\cos(nwt - nt_0)$$

What About Power?



ENERGY

v and i are expected to be periodic.

What About Power?

Assume a voltage

$$v(t) = \sum c_n \cos(n\omega t + \theta_n)$$

and a current

$$i(t) = \sum d_m \cos(m\omega t + \varphi_m)$$

with the same base frequency ω.

What About Power?

- We are interested in conversion:
  - Energy flow over time.
  - Determined by the average power flow <p(t)>
- Since $\omega = 2\pi/T$, then $1/T = \omega/(2\pi)$.

The Power Integral

$$p(t) = v(t) \cdot i(t)$$

$$p(t) = \left[\sum c_n \cos(\ )\right]\left[\sum d_m \cos(\ )\right]$$

Energy conversion

$$\langle p(t) \rangle = P_{ave}$$

$$= \frac{1}{T}\int_0^T \left[\sum c_n \cos(\ )\right]\left[\sum d_m \cos(\ )\right] dt$$

$$P_{ave} = \frac{1}{T}\int_0^T \sum_n \sum_m c_n d_m \cos(n\omega t + \theta_n)\cos(m\omega t + \phi_m) dt$$

Simplify

- Integration is a linear operation, so an integral of sums is the sum of the integrals.

$$P_{ave} = \frac{1}{T}\sum_{n}\sum_{m}\int_{0}^{T} c_n d_m \cos(n\omega t + \theta_n)\cos(m\omega t + \phi_m)\,dt$$

The Power Integral

- Sine and cosine have the following property:

$$\int_0^{2\pi} c_n d_m \cos(n\theta)\cos(m\theta)\, d\theta = 0 \quad \text{if } n \neq m$$

- Cross-frequency terms with n and m not equal do not contribute to average power.
- Average power becomes the sum of contributions at each frequency.

The Power Integral

- Only Fourier components that appear in both the voltage and the current will contribute to the average power flow.

$$P_{ave} = \frac{1}{T}\sum_{n=0}^{\infty}\int_{0}^{T} c_n d_n \cos^2(n\omega t + \ldots)dt$$

$$= c_0 d_0 + \frac{c_1 d_1}{2}\cos(\theta_1 - \phi_1) + \ldots$$

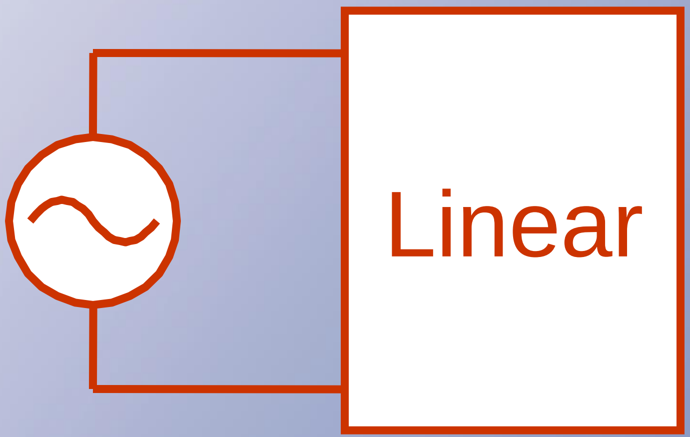
Frequency Matching

- *Frequency matching condition*:
  - To draw power from a source or
  - To deliver it to a load, there must be components at matching frequencies.
- If the source is given, we must match it.

Frequency Matching

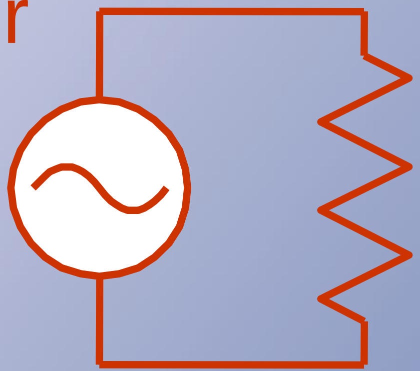
Source

$$V_0 \cos(120\pi t)$$



Must provide current at same frequency.

E.g.: Resistor



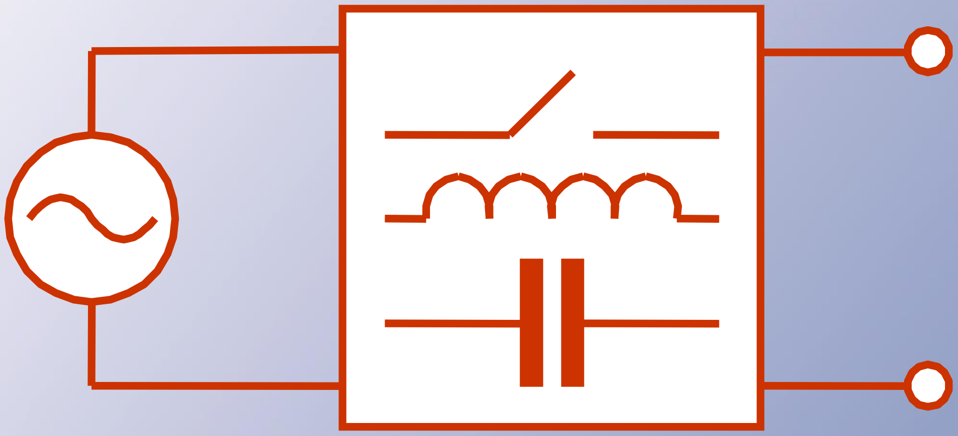
$$i(t) = \frac{v(t)}{R}$$

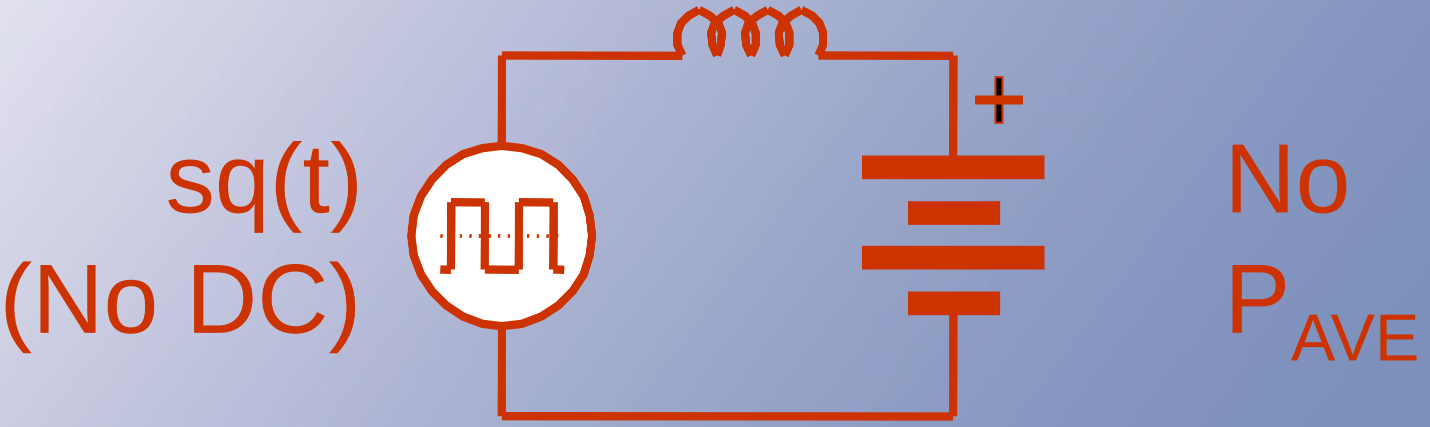
Some Examples

- 60 Hz to 50 Hz, voltage to current, conversion.
  - The converter creates the input current and the output voltage.
- The input current must have a 60 Hz component. The output voltage must have a 50 Hz component.
- Notice the implication of a *wanted component* associated with energy.

Frequency Matching

Example:

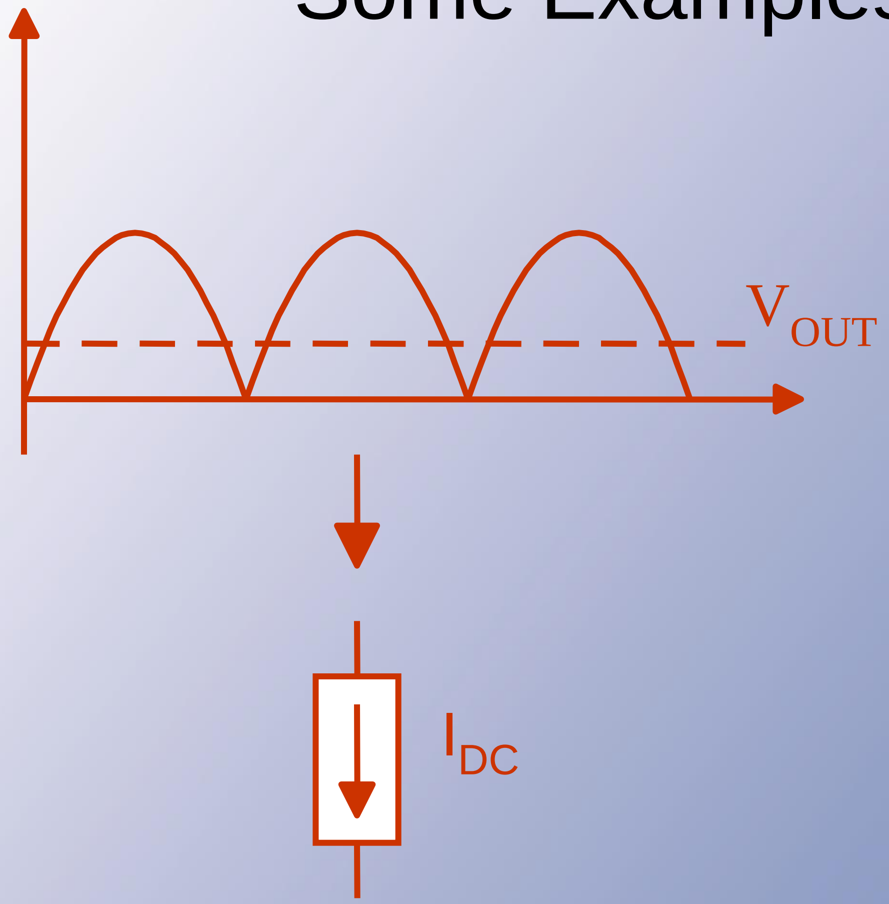




Some Examples

- If ac voltage is imposed on a battery, no average power will be delivered to the battery.
- In a rectifier, only the dc component of the output matters for power transfer.

Some Examples

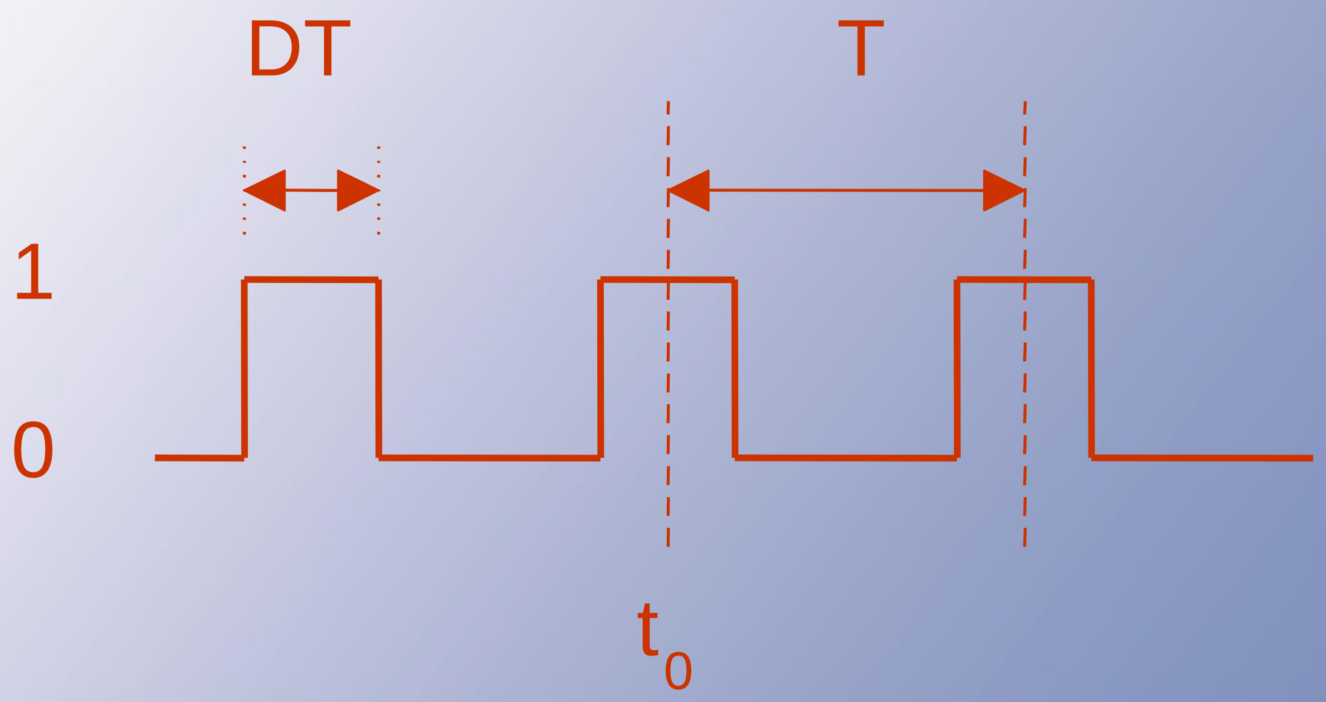


We need <V_{out}>, towards the power flow.

Critical Issues So Far

1. KVL + KCL
2. Polarity issues
   a) Restricted switch
3. Trial method
   b) Diode analysis
4. Frequency matching condition (i.e. *wanted* component)

Switching Functions

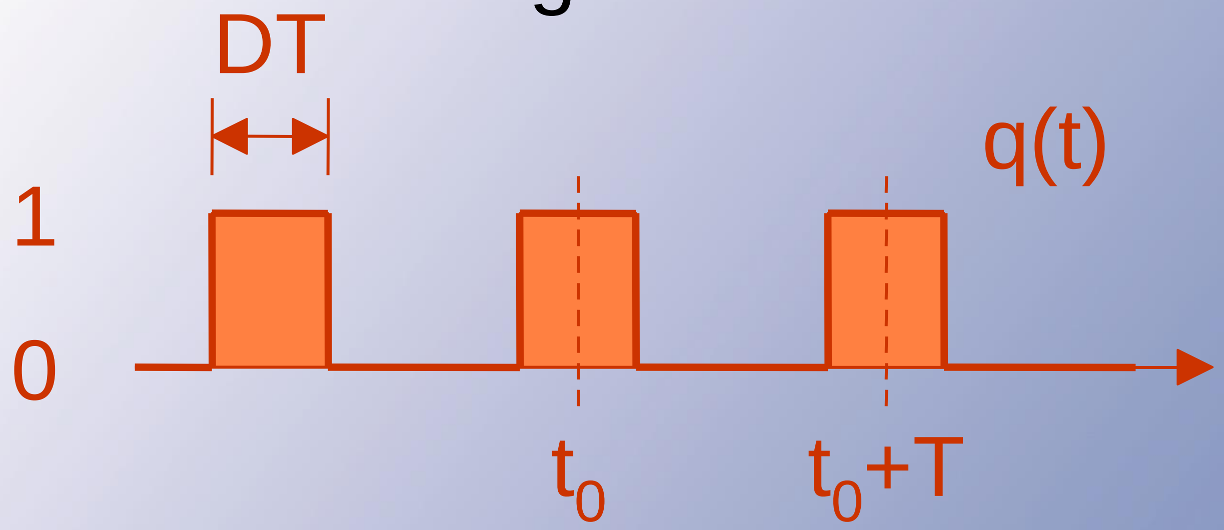


A general picture

Switching Functions

- This is a generalized switching function *q(t)*.
- We should perform Fourier analysis on this waveform.
- The average is *D*, the *duty ratio* or *duty cycle*.
- The frequency $f = 1/T$, or radian frequency $\omega = 2\pi/T$.

Switching Functions



$$\text{Average}\ \frac{1}{\mathrm{T}}\int_{t_0}^{T+t_0} q(t)\,dt = \frac{DT}{T} = D$$

“Duty Ratio”, or “Duty Cycle”

D=0 to 1, 0% to 100%

Switching Functions

- There is also a phase, determined by the time axis position t_0.
- We define a phase value $\phi_0 = \omega t_0$.

Switching Functions

$$f = \frac{1}{T}$$

$$\omega = \frac{2\pi}{T} = 2\pi f$$

$$a_n = \frac{2}{T} \int_{t_0}^{t_0+T} q(t)\cos(n\omega t)\,dt = \ldots$$

$$q(t) = D + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{\sin(n\pi D)}{n} \cos[n\omega(t - t_0)]$$

Switching Functions

$$q(t) = D + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\sin(n\pi D)}{n}\cos(n\omega t - \theta_n)$$

$$\theta_n = n\omega t_0$$

Fourier series of “generic” q(t)

Switching Functions

- This series involves three parameters.
- In general, a periodic (and single-pulsed) switching function is determined by:
  - Duty ratio (fraction of time when on)
  - Frequency
  - Phase or timing

Switching Functions

- To control a converter, we must manipulate switching functions.
- This means only three general methods are possible:
  - Duty ratio adjustment
  - Frequency adjustment (but must satisfy matching)
  - Phase adjustment

Switching Functions

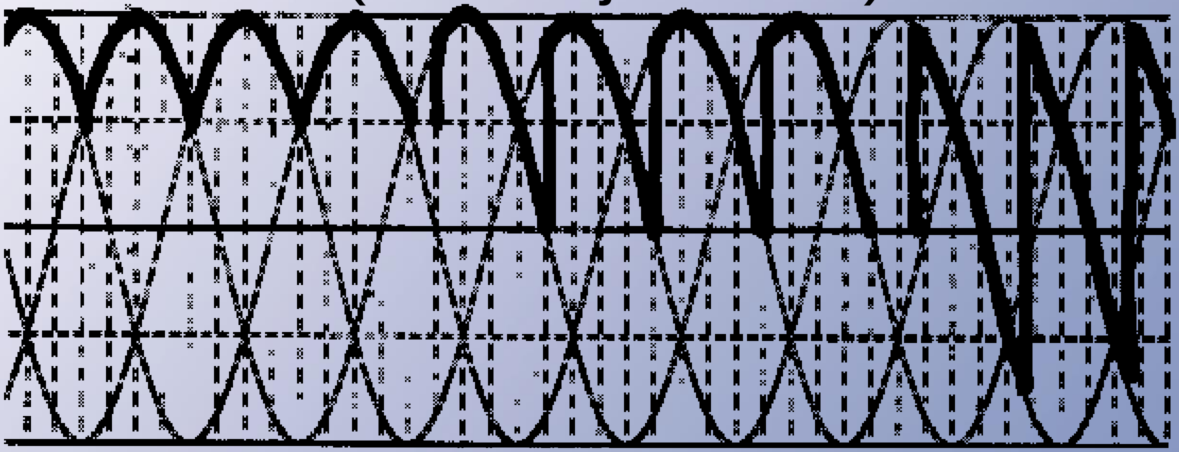
- Duty ratio:
  - Slow adjustment is called *duty ratio control.*
  - Or modulate it to vary regularly: pulse-width modulation (PWM)

Switching Functions

- Frequency:
  - Must meet matching conditions, so frequency adjustment is not common.
  - Frequency modulation is possible in principle but is rarely a good approach for power conversion.

Switching Functions

- Phase:
  - Phase control (slow adjustment)



  - Phase modulation (regular variation)

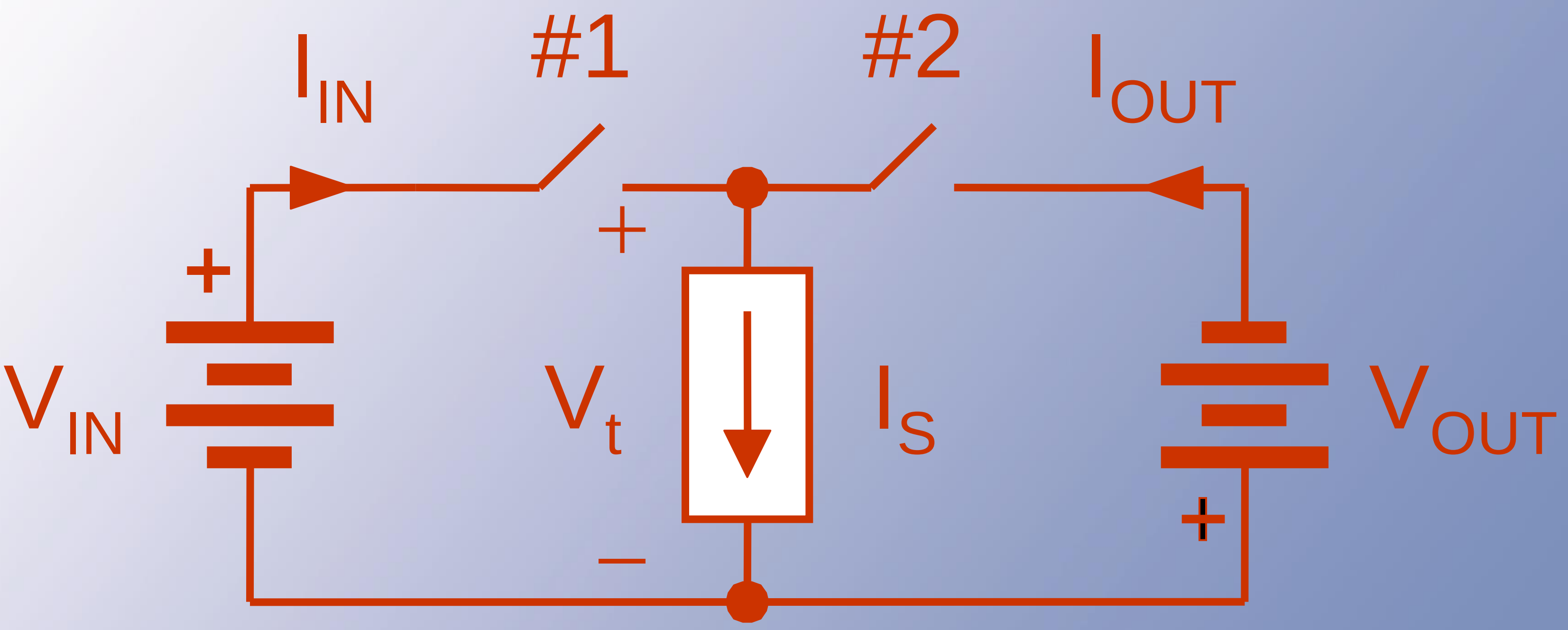
Source Conversion

- Concept: Converters transfer energy among ideal sources.
- Ideal voltage supplies any current, and no external effect can alter its voltage.
- We must match frequency to get nonzero energy flow over time.

Source Conversion

- Ideal current sources can deliver into any voltage. No external effect can alter the flow.
- A transfer source is an internal converter port with source characteristics.

Transfer Source Example



The current source in this converter is a transfer source.

Summary

- Non-zero average power appears only for matching Fourier component frequencies of *v* and *i*.
- There is a frequency matching condition for power flow.
- Switching functions: duty, frequency, and phase adjustment.
- Sources are governed by frequency matching requirements.

Distortion is Fundamental

- We generate waveforms that are piecewise sinusoids.
- They follow expressions such as $q(t)V_0 \cos(\omega t)$.
- This produces Fourier series results.

Distortion is Fundamental

$$v_{out}(t) = V_0 \cos(\omega t) \cdot \frac{2}{\pi} \sum \frac{\sin(n\pi D)}{n} \cos(n\omega t)$$

- There are terms $\cos(\omega t) \cdot \cos(n\omega t)$
- By trig identities (p. 699): $\cos[(n+1)\omega t] + \cos[(n-1)\omega t]$

Distortion is Fundamental

- The series represents an infinite number of terms and frequencies.
- We must design so the wanted one appears.
- There are infinite unwanted terms.

Distortion is Fundamental

- Distortion is fundamental:
  - There will always be unwanted terms in addition to wanted terms.
  - A switching converter does not produce perfect waveforms (ac or dc).

Distortion is Fundamental

- We must accept unwanted terms -- *distortion* -- in exchange for a lossless switching process.
- It is a question of degree: we would hope for low distortion, but it cannot be zero.

Measures

- Since distortion must be present, we need to characterize or measure it.
- For dc output, the distortion is collected ac terms, called *ripple*.
- For ac output, we can talk about harmonics (harmonic distortion).

Dc Measures

- In typical dc applications, ripple is about 1% (although it is hard to achieve less than about 50 mV).
- The usual measures are either the peak-to-peak or RMS values of the waveform, less its dc component.

Dc Measures

- This is only part of the story.
- Ripple in the audio band is often considered especially objectionable.
- Ultrasonic ripple can be a problem in some applications.

Ac Measures

- How much of the signal is harmonics?
- *Total harmonic distortion* (THD) measures the distortion content as a fraction of the fundamental.

$$THD = \sqrt{\frac{\sum_{n=2}^{\infty} c_n^2}{c_1^2}}$$

Harmonic Distortion

- *Total harmonic ratio* (THR) measures the distortion content as a fraction of the RMS value.
- *Total unwanted distortion* (TUD) measures the distortion content as a fraction of the RMS value of the wanted harmonic.
- THD is used most often.

Harmonic Distortion

- THD has no upper limit. Here are two guidelines:
  - Waveforms with THD below about 1% look sinusoidal on an oscilloscope.
  - Most converter waveforms are more distorted.
- The THD or TUD value can exceed 100% (when no filtering is used).
- Filters can reduce but not eliminate harmonics.

Harmonic Distortion

- THR values cannot exceed 100%, based on the definition.
- When distortion is “low” (below about 50%), the THD and THR values are not far apart.

$$THR = \sqrt{\frac{\sum_{n=2}^{\infty} c_n^2}{\sum_{n=1}^{\infty} c_n^2}}$$

Harmonic Distortion: TUD

- Often, the fundamental is not the wanted component.
- In this case, THD and THR are of no interest.
- We can define a *total unwanted distortion* (TUD) value, a ratio of unwanted to wanted.

Harmonic Distortion

Total unwanted distortion (TUD):

n (wanted) (n $\geq$ 1)

$$TUD = \sqrt{\frac{\sum_{n \neq n_{wanted}}^{\infty} c_n^2}{c_{n_{wanted}}^2}}$$

Computing THD

- THD often is not hard to compute, because the RMS value of a periodic waveform is

$$f_{RMS} = \left(\tfrac{1}{2} \sum c_n^2\right)^{1/2}$$

- If we know the RMS value and also c_1, we can find the total harmonics.

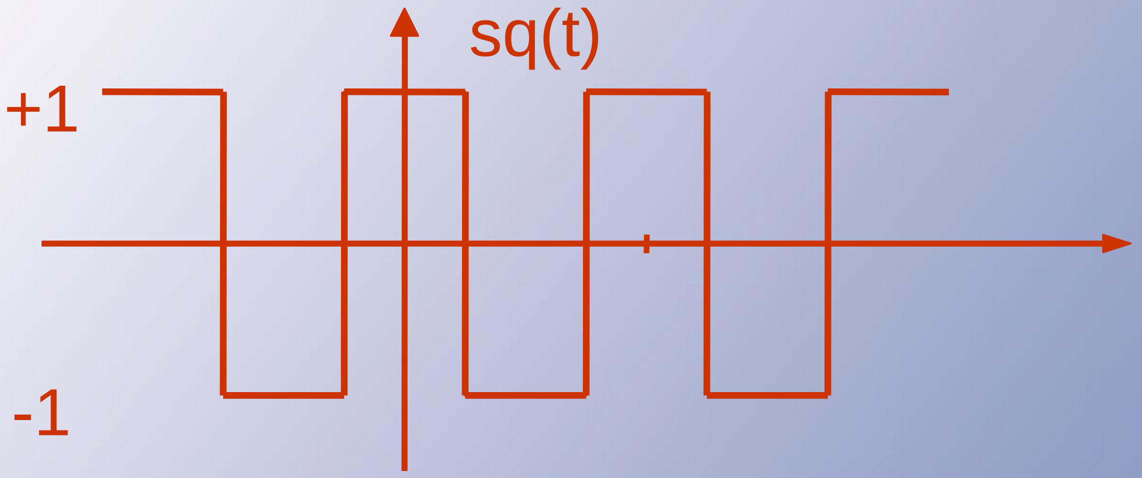
Alternative THD Expression

Periodic with no DC: RMS $= \sqrt{\frac{1}{T}\int_0^T f^2(t)\,dt} = \sqrt{\frac{1}{2}\sum_{n=1}^{\infty} c_n^2}$

$$THD = \sqrt{\frac{\sum_{n=2}^{\infty} c_n^2}{c_1^2}} = \sqrt{\frac{2 \cdot RMS^2 - c_1^2}{c_1^2}}$$

The RMS value, together with c_1, lets us compute the THD.

THD Example



$$THD = \sqrt{\frac{\sum_{n=2}^{\infty} c_n^2}{c_1^2}}$$

Alternative THD Expression

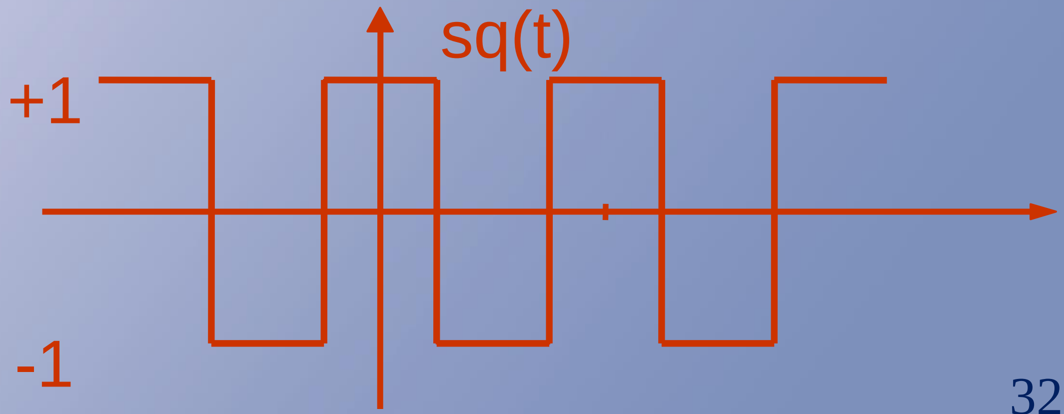
$$2(RMS)^2 = \sum_{n=1}^{\infty} c_n^2$$

$$= c_1^2 + c_2^2 + \ldots$$

$$2(RMS)^2 - c_1^2 = \sum_{n=2}^{\infty} c_n^2$$

RMS → 1

$$c_1 = 4/\pi$$



Alternative THD Expression

$$\sqrt{\frac{2\cdot RMS^2 - c_1^2}{c_1^2}} = \sqrt{\frac{2 - 16/\pi^2}{16/\pi^2}} = \sqrt{\frac{\pi^2 - 8}{8}} = 0.483$$

THD (sq(t)) = 0.483

48.3%

THR (has 2•RMS^2 in denominator)

THR = 0.435

43.5%

Examples

- The square wave sq(t) has THD of 48.3% and THR of 43.5%.
- The triangle wave has THD of 12.1%.

Examples

- More general ac-ac conversion waveforms exceed THD values of 100%.

Regulation

- Ripple and harmonics tell us about distortion.
- We also want to know how closely an ideal source is approached.

THD, TUD

→ Distortion (harmonics)

→ “Source quality”

Real voltage source → change

Regulation

- *Regulation* is a set of measures that tell us how “ideal” a real source will be.
- An ideal source never changes, so regulation measures change.
- Ideally, regulation values are 0.

Regulation

- The most general measures are partial derivatives, such as:

$$\frac{\partial V_{out}}{\partial V_{in}}$$

- This is not so useful, since we do conversion. A better ratio is:

$$\frac{\partial V_{out} / V_{out}}{\partial V_{in} / V_{in}}$$

Regulation

- Change is taken with respect to any variable of interest.
- Example: a dc output, V_{out}.
- Ripple is one thing. We also want to know how the dc value changes:

$$\partial V_{out} / \partial (\ )$$

- The variable could be input voltage, load current, time, temperature, …

Regulation

- In most cases, relative change is needed.
- Example: 120 V to 1 V and 5 V to 1 V converters.
- If $\frac{\partial V_{out}}{\partial V_{in}} = 1\%$ what does this mean?
- This measure is absolute, but not useful.

Regulation

- Relative change.
- Correct but not often used:

$$\frac{\partial V_{out} / V_{out}}{\partial V_{in} / V_{in}} = x$$

- Usually written:

$$\frac{\partial V_{out} / \partial V_{in}}{V_{out} / V_{in}} = x$$

1% input change → 1% output change

Regulation

Relative value 1 → No Regulation

Ideally → 0

Regulation

- Most products measure regulation in terms of a specific change rather than in terms of a partial derivative.
- A typical value is $\Delta V_{out}/V_{out(nom)}$, for some specified change in conditions.

Regulation

- Line regulation: $$\frac{\Delta V_{out}(V_{in})}{V_{out(nom)}}$$
- V_{in} is taken over the allowed range of input values.
- Sample for a converter with 120 V RMS input (±10%) and 1 V dc nominal output: Check output with 132 V input, 108 V input, and other values in between.

Line Regulation Example

- The regulation value is

$$\left.\frac{V_{out(\max)} - V_{out(\min)}}{1\text{V}}\right|_{allowed\ line\ values} = \text{Line reg.}$$

- Checked over the allowed range of input values.
- Usually expressed in %.
- A value of 0.1% would require a total deviation of less than 1 mV for this converter.

Regulation

- Values of interest:
  - Line regulation, change in output when the input is altered
  - Load regulation, change in output as the load current or power changes
  - Temperature regulation
  - Time regulation (drift)

Regulation: Example

- Consider a resistive voltage divider with no load.
- This provides an output proportion to the input.
- A derivative line regulation measure gives 1, or 100%.
- This means that any input change appears directly at the output.

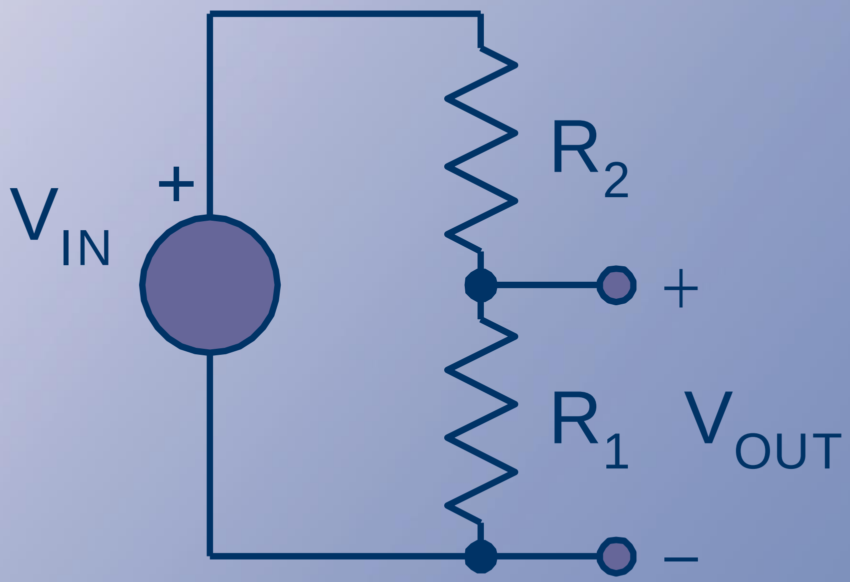
Regulation

- Voltage divider

$$V_{out} = \frac{R_1}{R_1 + R_2} V_{in}$$

$$\frac{\partial V_{out}}{\partial V_{in}} = \frac{R_1}{R_1 + R_2}$$

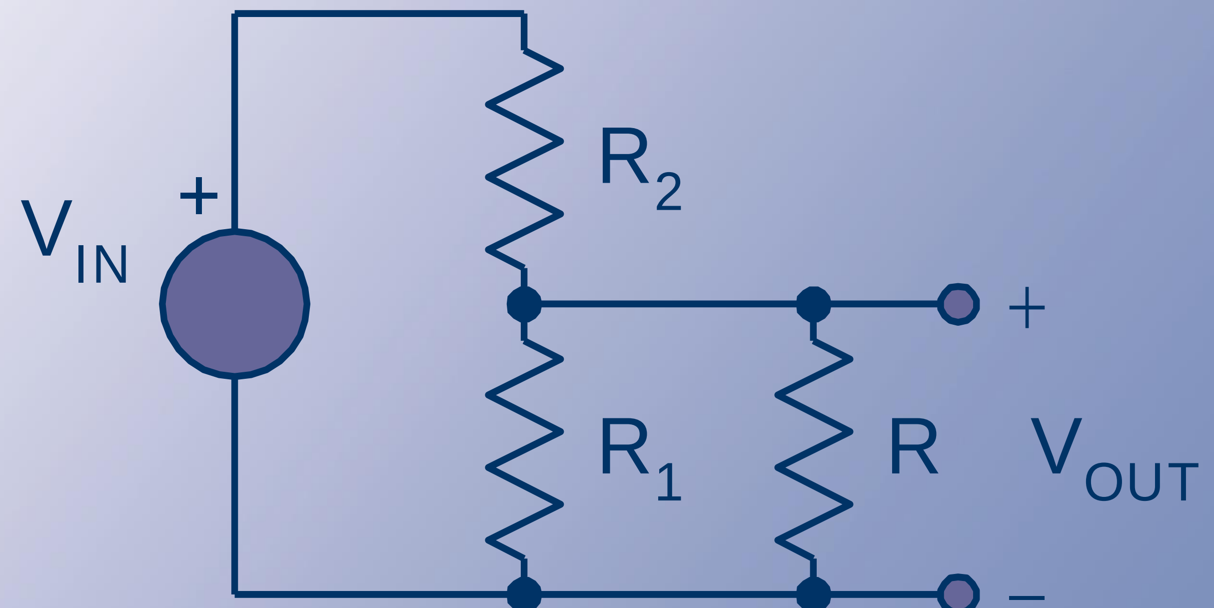
$$\frac{\partial V_{out} / \partial V_{in}}{V_{out} / V_{in}} = 1 \quad \Rightarrow$$



Unregulated!

Regulation

Voltage divider: Even worse with a load.
Now the line regulation is above 100%!



Regulation

- Modern dc supplies often have source regulation at the 0.1% level.
- Load regulation can be more difficult: it depends on wiring.

Summary So Far

- We seek ideal sources, but distortion and variation must appear: *wanted* vs. *unwanted* components.
- THD and ripple measure distortion.
- Regulation measures tell us about the wanted component portion.

Regulation Examples

Example:

DC supply with 0.1% line regulation.

Input: 120 V_{rms} 60 Hz.

Output: 5 V.

Assume input = 120 V ± 10% (108 V to 132 V)

$$\frac{\Delta V_{out}\big|_{V_{in}}}{V_{out}^{nom}} < 0.001$$

Regulation Examples

- With input variation as allowed, the output change will not exceed 5 mV.
- Ripple (which is about 50 mV), is not included in the line regulation definition.

Regulation Examples

Example:

Input: 85 to 265 V_{rms} 60 Hz

Output: $V_{out,nom}$=12 V

Vin RMS (V)	Vout DC (V)
85	12.032
95	12.027
105	12.052
115	12.058
205	12.069
220	12.073
240	12.072
265	12.075

Regulation Examples

Line regulation:

$$\Delta V_{out} = 12.075 - 12.027$$

$$= 0.048V$$

(worst case)

$$\frac{\Delta V_{out}}{V_{out}^{nom}} = \frac{0.048}{12}$$

$$= 0.004$$

$$= 0.4\%$$

Load Regulation

Definition? $\frac{\partial V_{out}}{\partial I_{out}}$ $\frac{\partial V_{out}}{\partial R_{load}}$ $\frac{\partial V_{out}}{\partial P_{out}}$ …

Typical: $\frac{V_{out}^{max} - V_{out}^{min}}{V_{out}^{nom}}$

Measured over the allowed load range

Load Regulation Example

Example:

Output: 5 V, 10 to 100 W

$$\Delta V_{out} = 4.992 - 4.989$$
$$= 0.003\,V$$

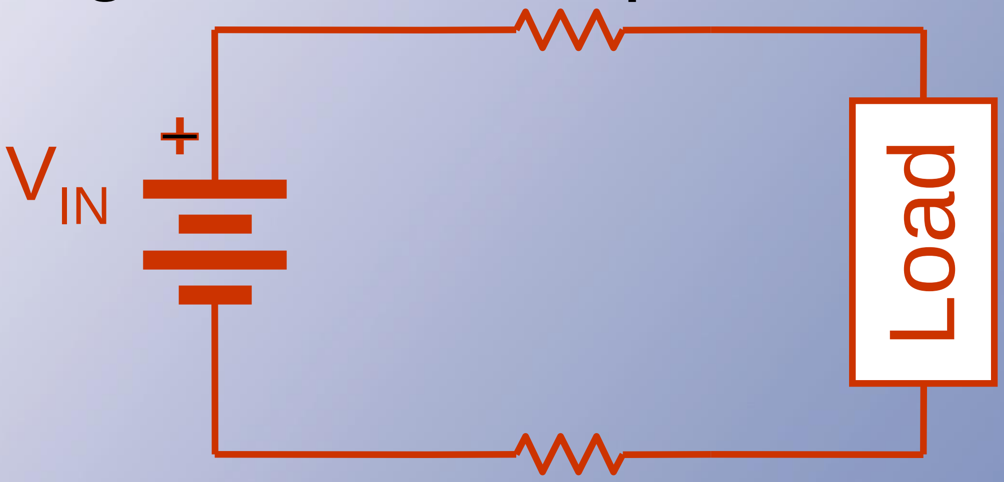
$$\frac{\Delta V_{out}}{V_{out}^{nom}} = \frac{0.003}{5}$$
$$= 0.0006 = 0.06\%$$

Pout (W)	Vout (V)
0	5.260
10	4.992
30	4.991
50	4.991
70	4.991
90	4.989
100	4.990

However, if we measure the load regulation in the lab with normal wires, we'll get higher values. Why?

Load Regulation Example

Reason:



10 W →I = 2 A

100 W →I = 20 A

0.06% change 3 mV

$$R = \frac{V}{I} = \frac{3\text{ mV}}{20\text{ A}} = 150\ \mu\Omega$$

Each wire must have 75 μΩ or less!

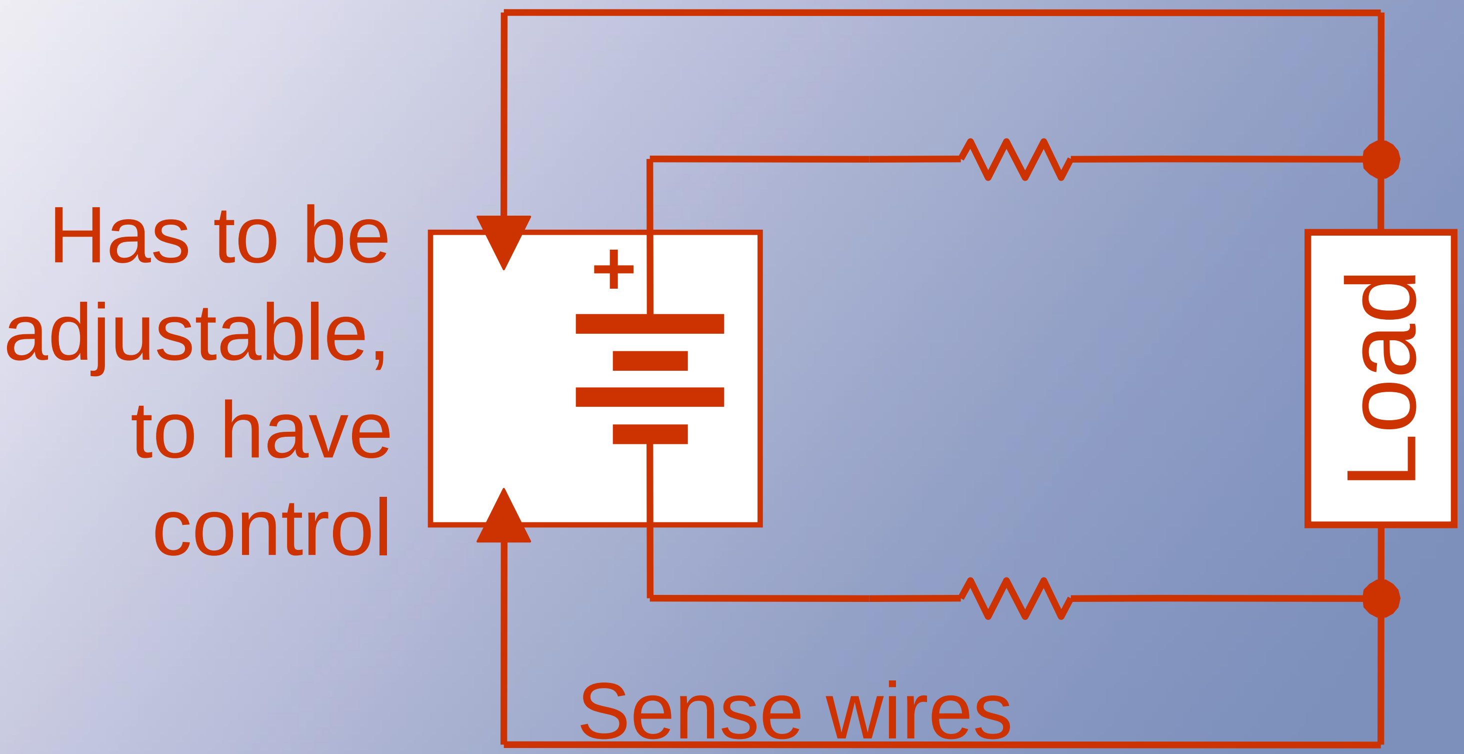
Very hard to accomplish!

Regulation

- Some power supplies are called regulators, if their main function is to prevent change at the output.
- Control is required to achieve good regulation.

Regulation Examples

Kelvin connection: Sense wires have I=0 or I=constant.



Regulation

Temperature regulation:

$$\frac{\partial V_{out}}{\partial T} \quad \text{or} \quad \frac{\partial V_{out}/V_{out}}{\partial T}$$

Example: 0.04% per °C.

Range: 20°C to 35°C (15°C change)

Output change ← 0.6%

Regulation

Example: Automotive systems

Range: -20°C to +50°C

(70°C change)

Output change less than

(0.04%) x (70°) =

2.8%