POWER ELECTRONICS

Key Concepts

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Notation Points

- Upper case for dc quantities. V_{in}
- Lower case for ac quantities. $v(t)$
- Angle brackets < > for averages.

$$\frac{1}{T}\int_0^T f(t)\,dt = \langle f(t) \rangle = F$$

Notation Points

- Time-varying average (moving average):

$$\frac{1}{T}\int_{t-T}^{t} f(\tau)\,d\tau = \bar{f}(t) = F(t)$$

$$f(t) = F(t) + \tilde{f}(t)$$

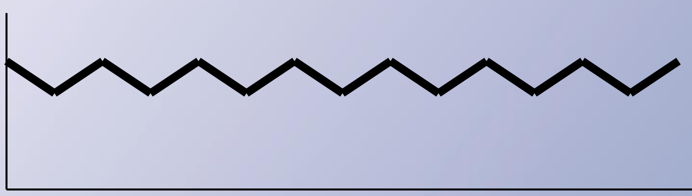
$$F(t): \text{Moving average}$$

$$\tilde{f}(t): \text{Time variation}$$

Time-Varying Average

(Moving average)

$$\frac{1}{T}\int_{t-T}^{t} f(\tau)dt = \bar{f}(\tau)$$



$\leftarrow f(t) = F + \tilde{f}(t)$

Notation Points

RMS (Root Mean Square):

$$F_{RMS} = \sqrt{\frac{1}{T}\int_0^T f^2(t)\,dt}$$

If $f(t) = V_0 \cos(wt)$ then $F_{RMS} = \frac{V_0}{\sqrt{2}}$

Power Electronic System

Electrical source.

Conversion.

Load.

Power Electronic System

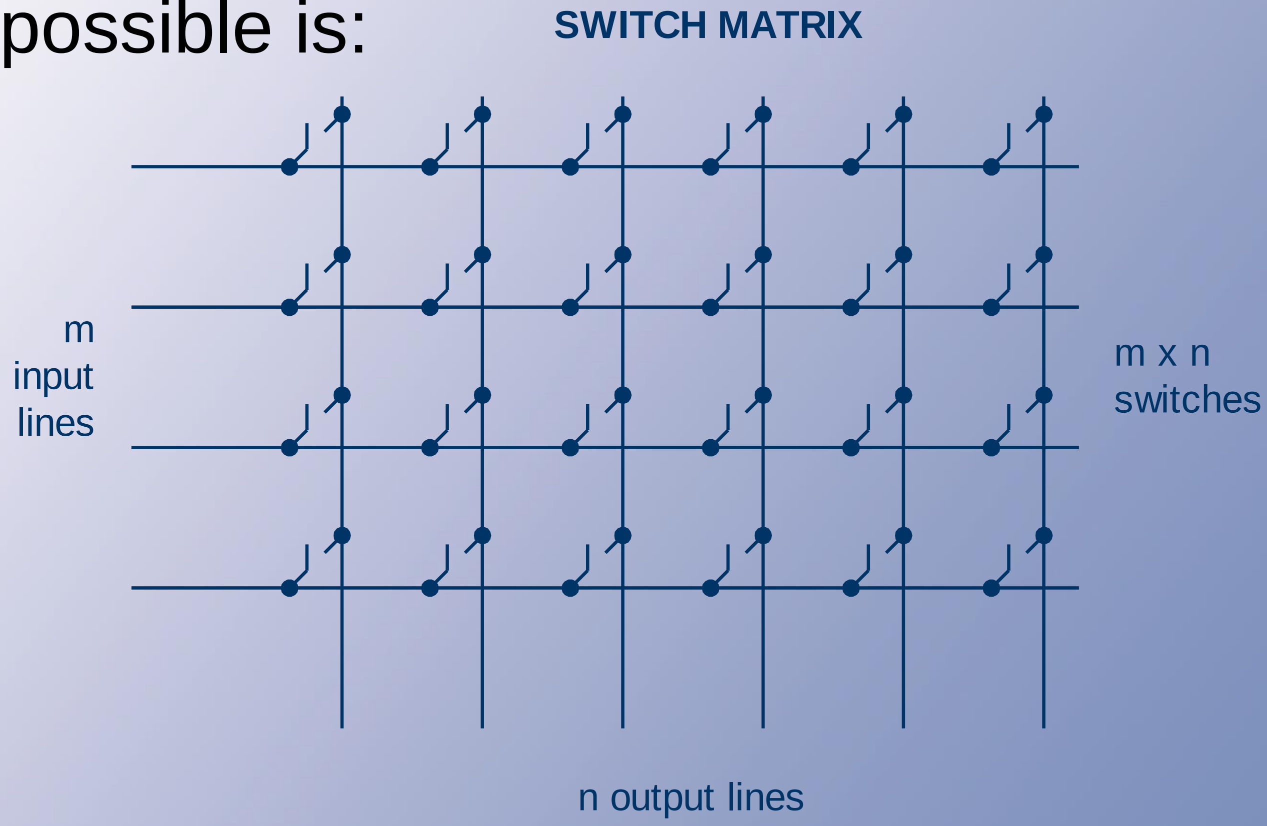
Conversion components

The Switch Matrix

If a converter has *m* input lines and *n* output lines, an *m* x *n* matrix allows all possible interconnections.

Conversion

- Consider switches alone (no storage elements). The most complicated arrangement possible is:

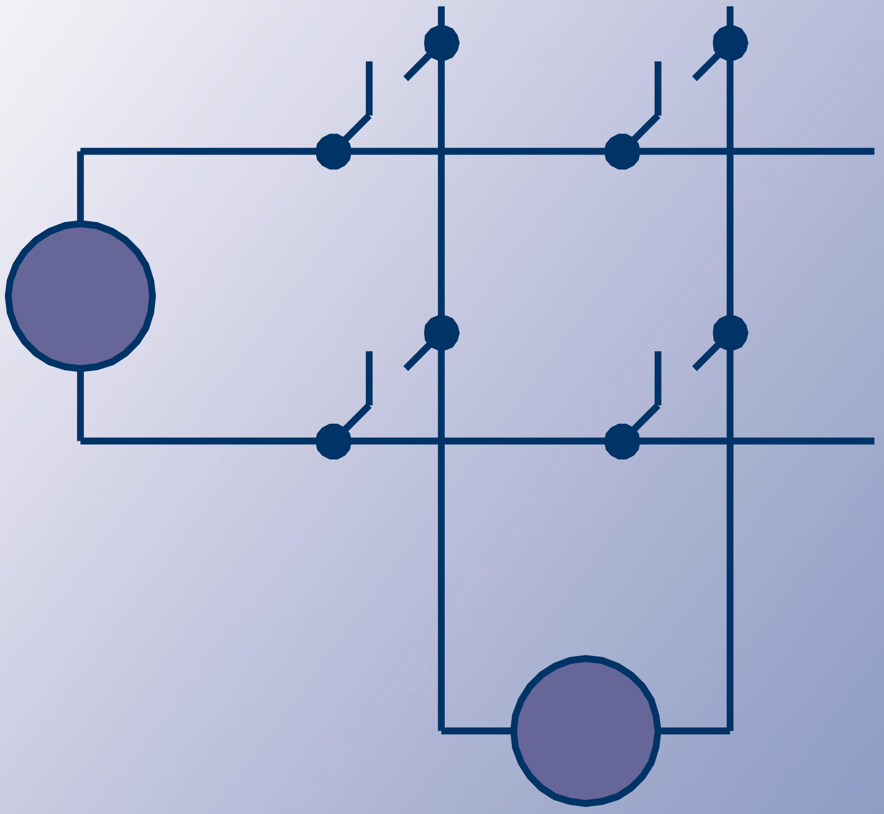


Conversion

- Polyphase case:
  - Three inputs, three outputs
- High-voltage dc:
  - Up to 48 or more input lines
  - Perhaps 6 output lines
- Typical case:
  - 2 x 2 matrix

Conversion

- Typical case with only 4 switches:



2 x 2 =
4 switches

Power Electronics Focus

1) Build a switch matrix.

Hardware.

2) Determine how to operate matrix to get a desired result.

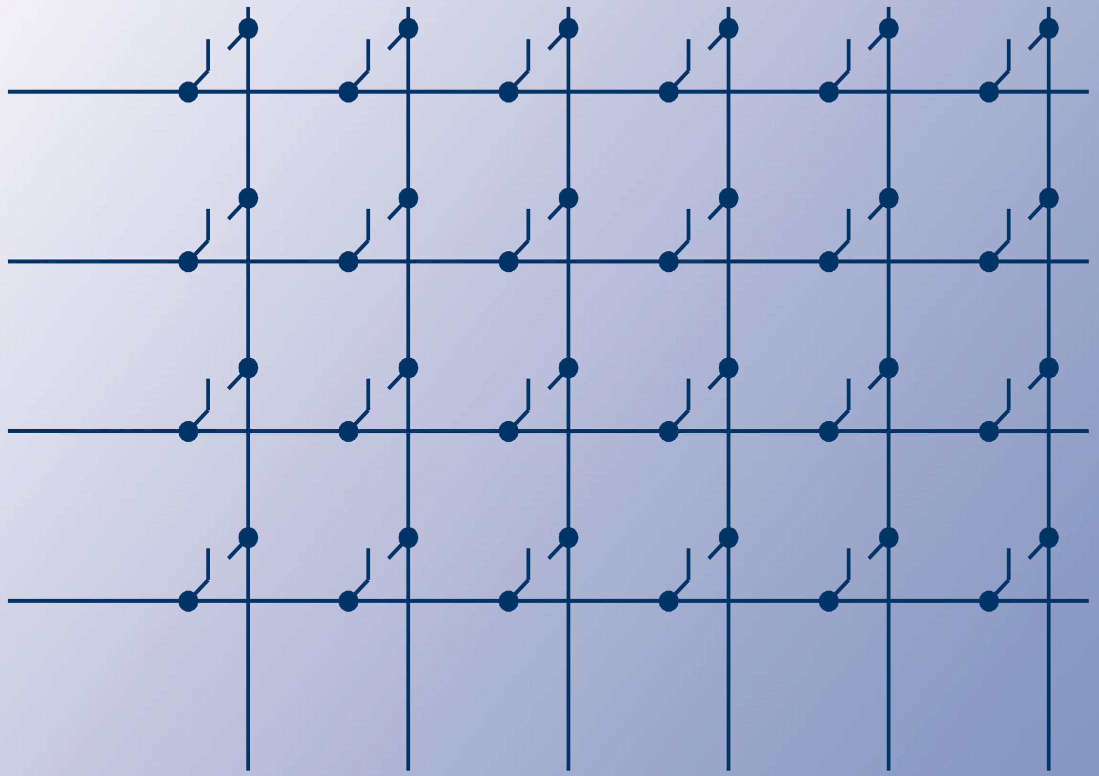
Software.

3) Use storage elements to interface with in/out.

Interface.

Conversion

m
input
lines



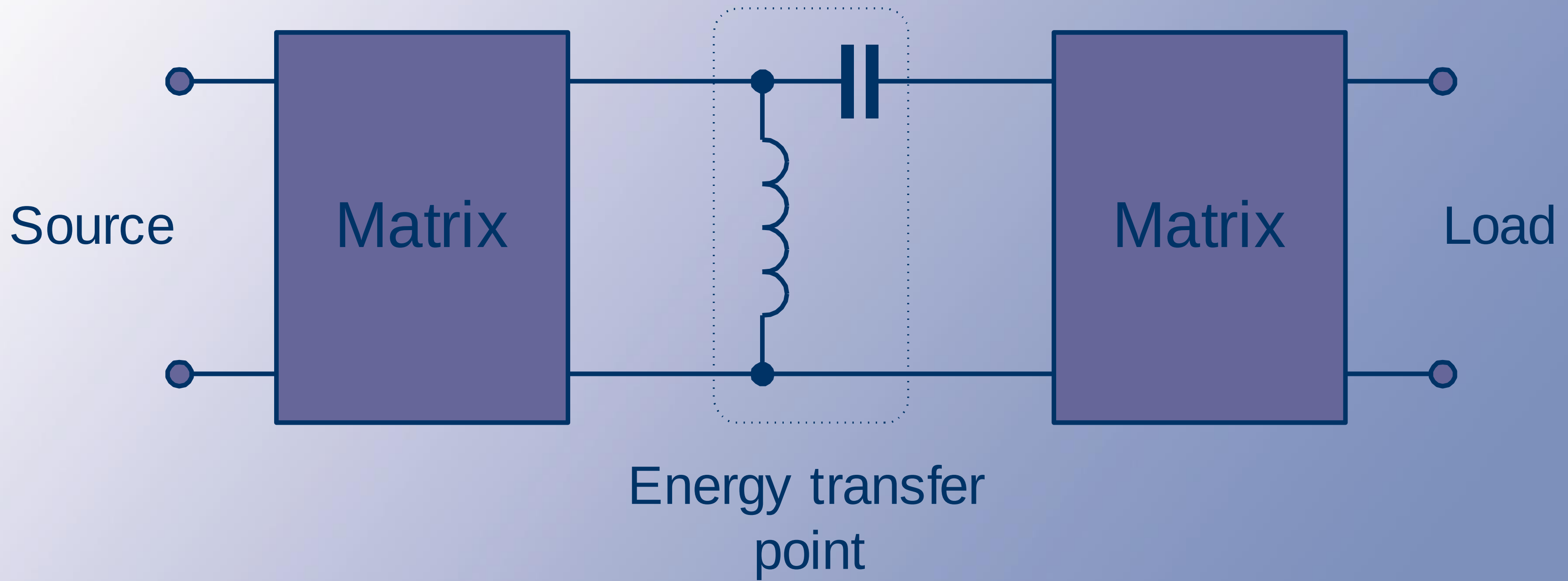
m x n
switches

n output lines

Direct switch matrix (No storage)

Conversion

Indirect switch matrix



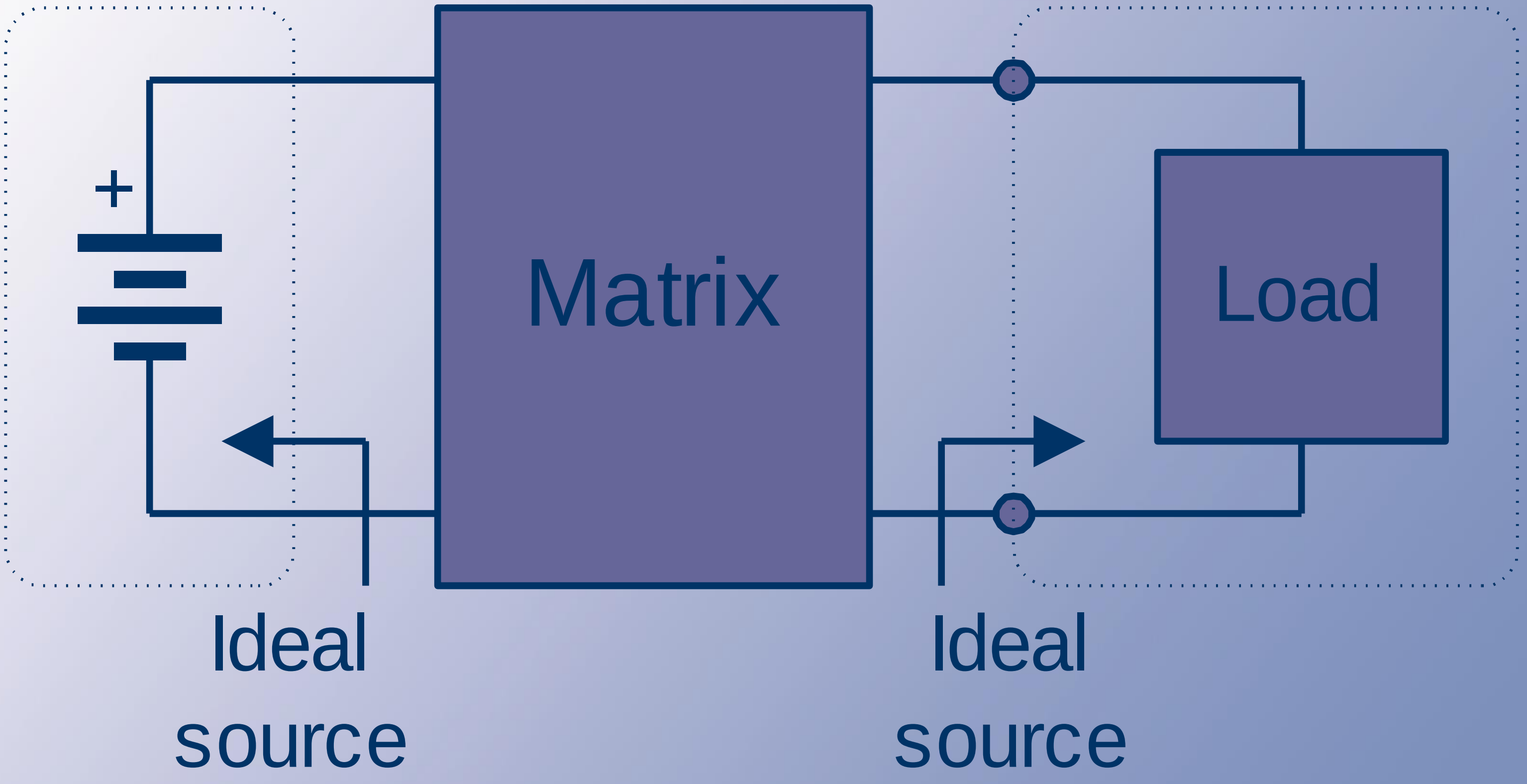
Methods

- The usual method for design and analysis of indirect switch matrix converters is to cascade two direct switch matrices and place the storage components between them.
- Cascaded converters are common:
  - Rectifier-inverter sets for motor drives
  - Rectifier-dc sets for power supplies
  - Dc-dc converter cascades for flexibility

Source Conversion

- Energy conversion is not a generic process.
- User expectations:
  - **Voltage source**.
- Generalization:
- Electrical input: (Ideal) **Source**.
- Electrical load: User wants ideal source.

Source Conversion

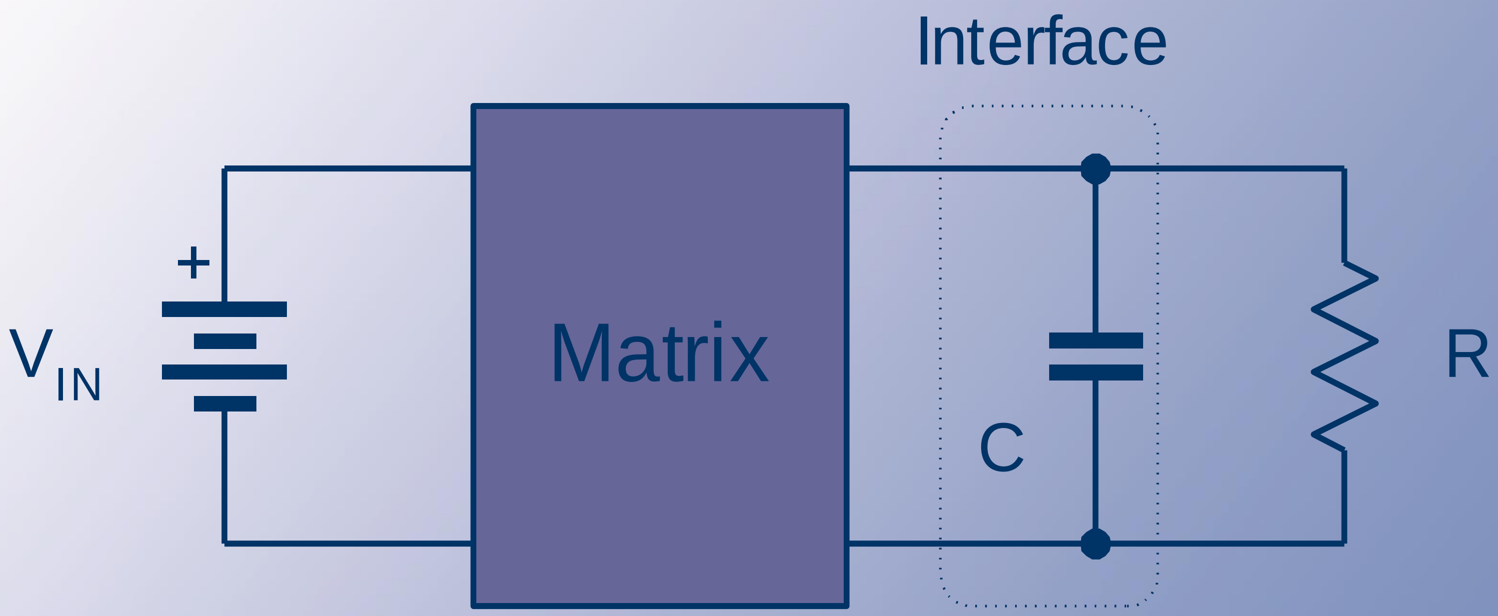


Source Conversion

Implications: We can analyze the circuit this way.

Power Electronic Circuit: Exchange energy among ideal sources.

Source Conversion

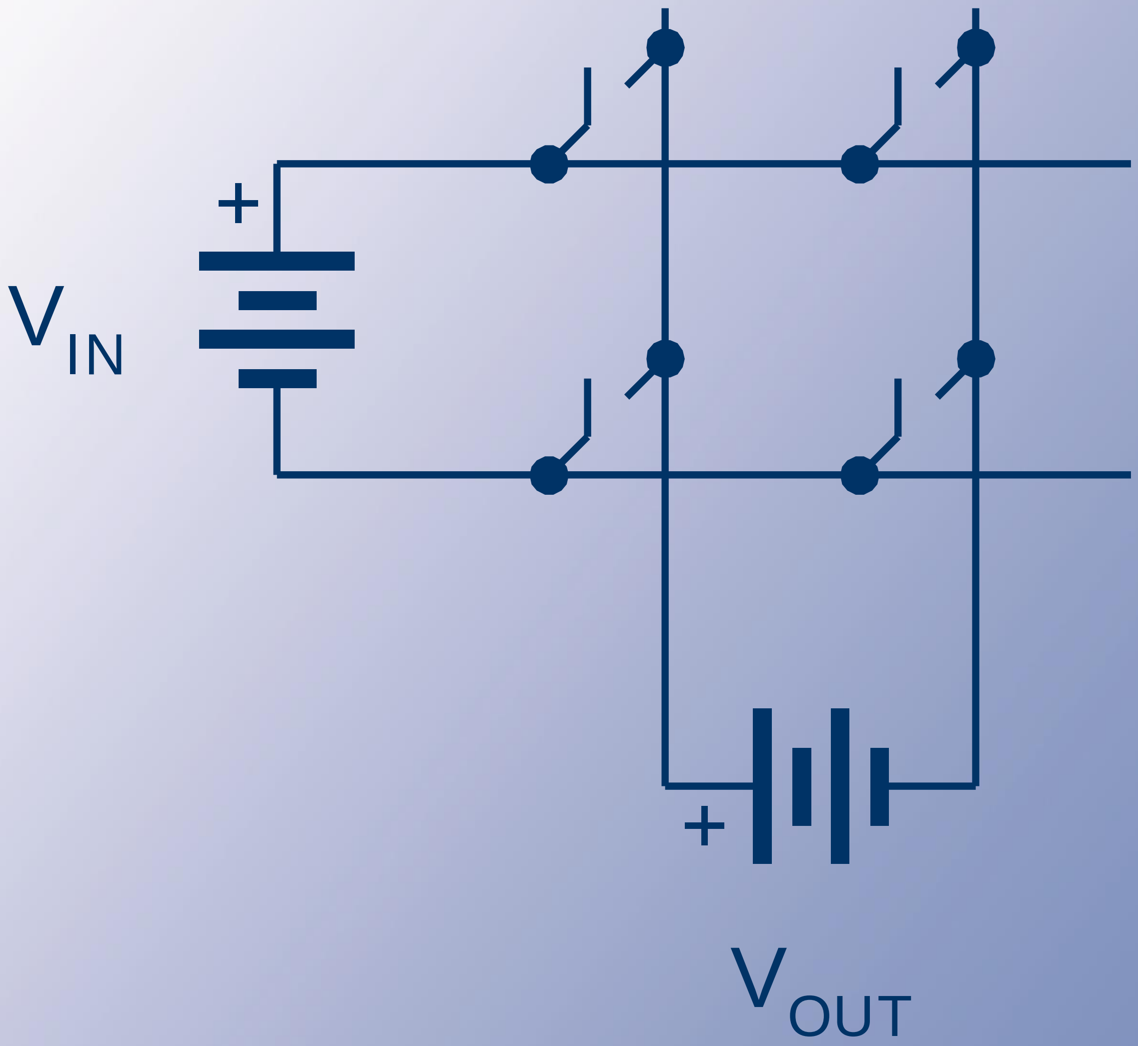


Large C ⇒ Little change in V_c (= V_{out}).

Source Conversion

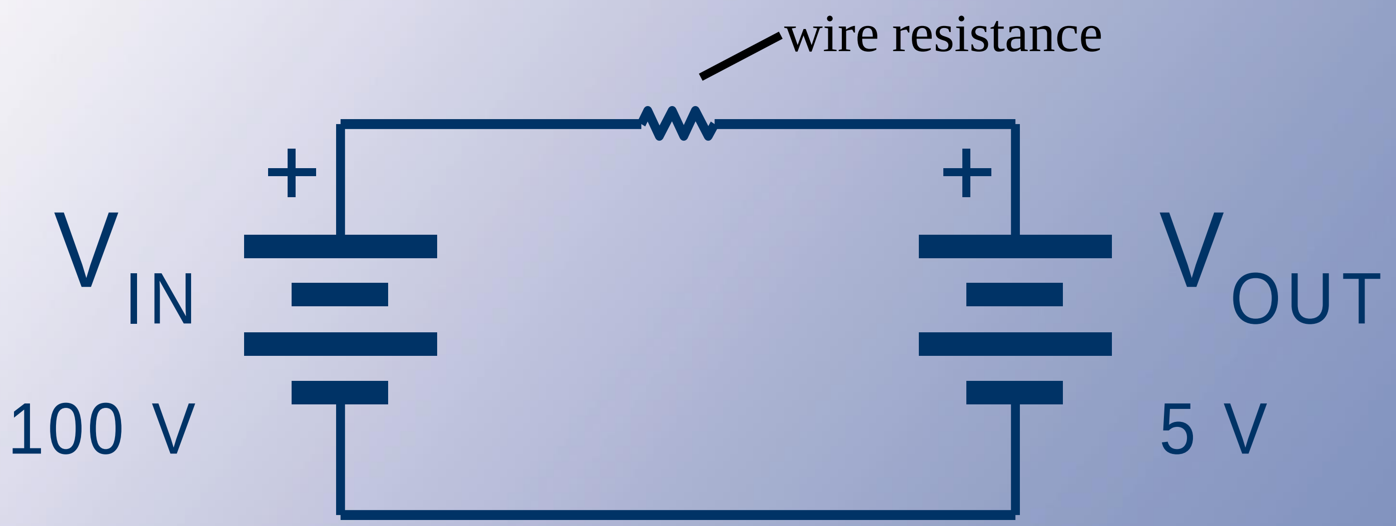
- The following slide shows a logical source conversion approach.
- The input and output are ideal dc voltages, and a switch matrix sits between them as a direct converter.
- Check the configurations.

Source Conversion



Source Conversion

One configuration would be:



Circuit laws:

KVL : $\Sigma V_{loop} = 0$

KCL : $\Sigma I_{node} = 0$

Circuit Laws

- There is a problem here!
- It would seem that the sum of voltages around the loop is nonzero!
- The reality is that wires and real devices have some (small) resistance.
- A large current will flow – and we hope will blow a fuse.

Circuit Laws

- KVL problem: Cannot interconnect unlike voltages.
- Trouble: switches do not “know” KVL.
- Power converter: Can **attempt** a violation.
- But a violation will not really occur – only a problem (or a fire).

The Reality of KVL

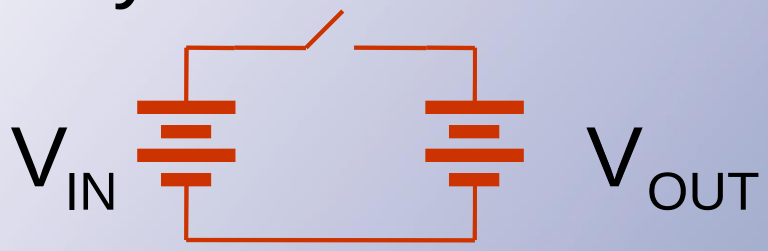
- We see that KVL has a concrete meaning in power electronics.
- The Law becomes: *Do not interconnect unlike voltage sources.*
- There is a real and often costly penalty for a violation.
- For design, we can think in terms like “Do not even *try* to violate KVL.”

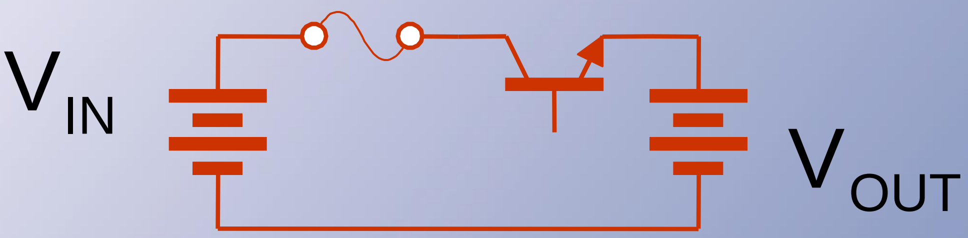
Reality of Circuit Laws

KVL: $\Sigma\, v_{loop} = 0$

- Implication: unequal voltage sources cannot be interconnected.
  - In power electronics (not in other fields), we can build a circuit that tries to "violate" KVL.
  - Attempts to connect unlike voltages yield extreme currents and failures.

Reality of Circuit Laws





- In the top circuit, the switch must be OFF so that KVL is not violated.
- In the bottom circuit, unfortunately, the transistor will probably fail before the fuse does anything.

Reality of Circuit Laws

KCL: $\Sigma\, i_{node} = 0$

- The implication is that unequal current sources cannot be interconnected.
- It is possible to build a circuit that tries to "violate" KCL.

Example: Inductor carrying current. Disconnect it abruptly!

Current flow continues

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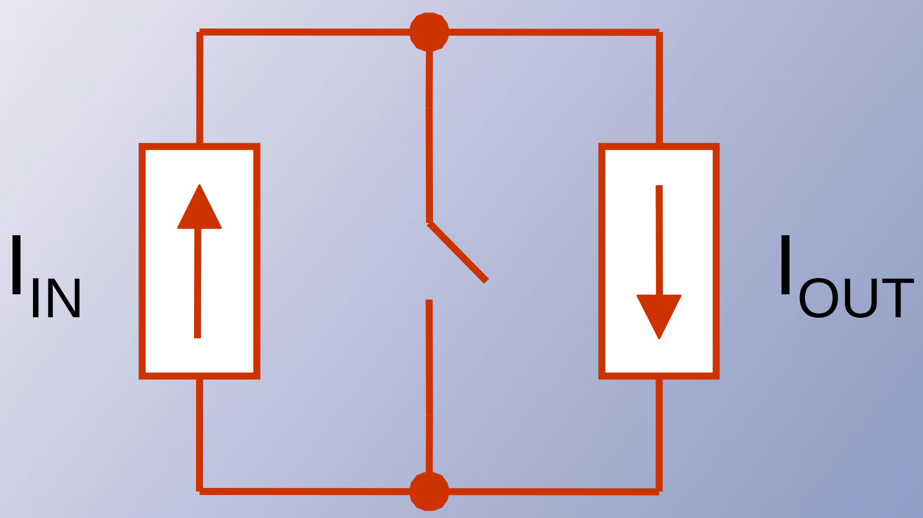
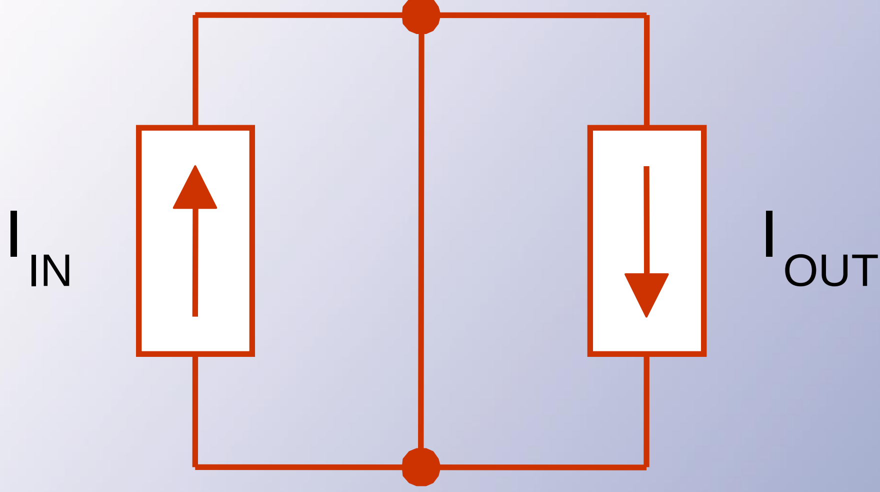
Reality of Circuit Laws

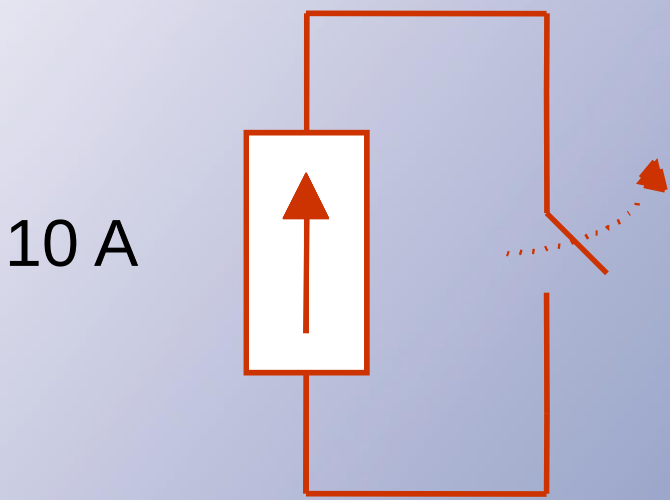


The switch must be ON, so that KCL is not violated.

Reality of Circuit Laws

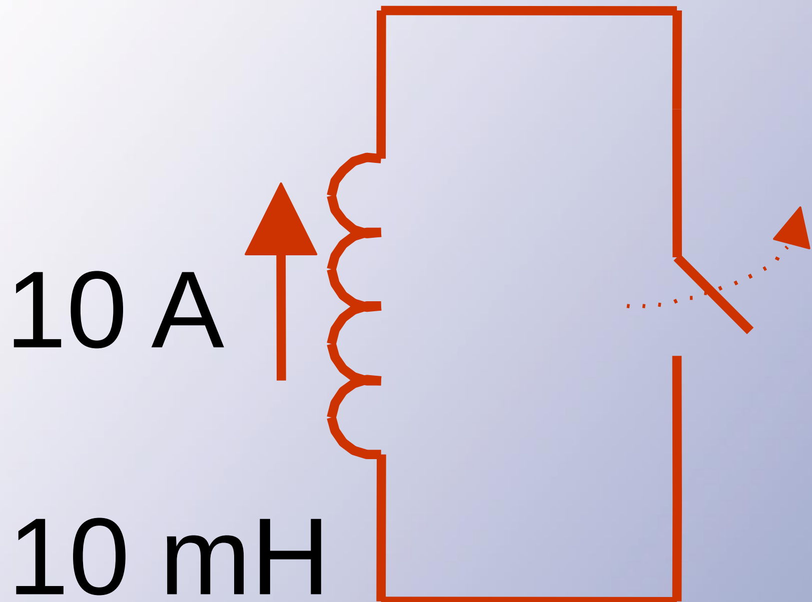


Fine!



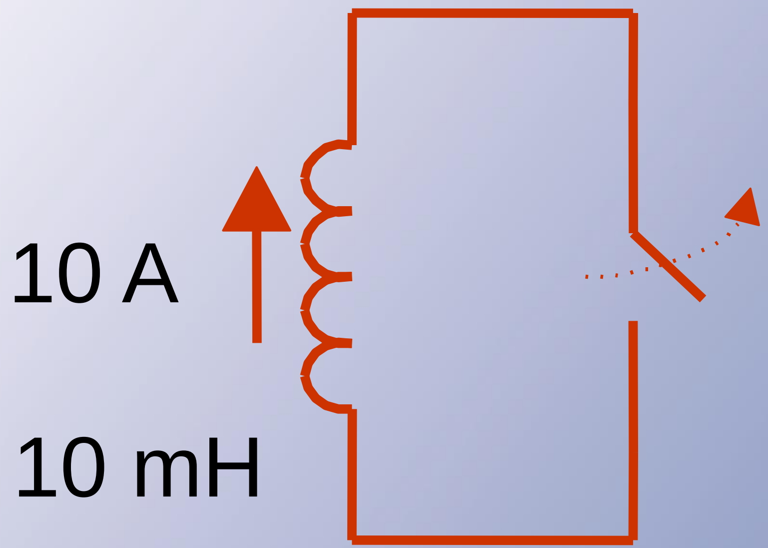
After opening the switch, the electrons must go somewhere. An arc will appear.

One-Port Model



v = L di/dt. If current drops in 1 µs, we have:

One-Port Model



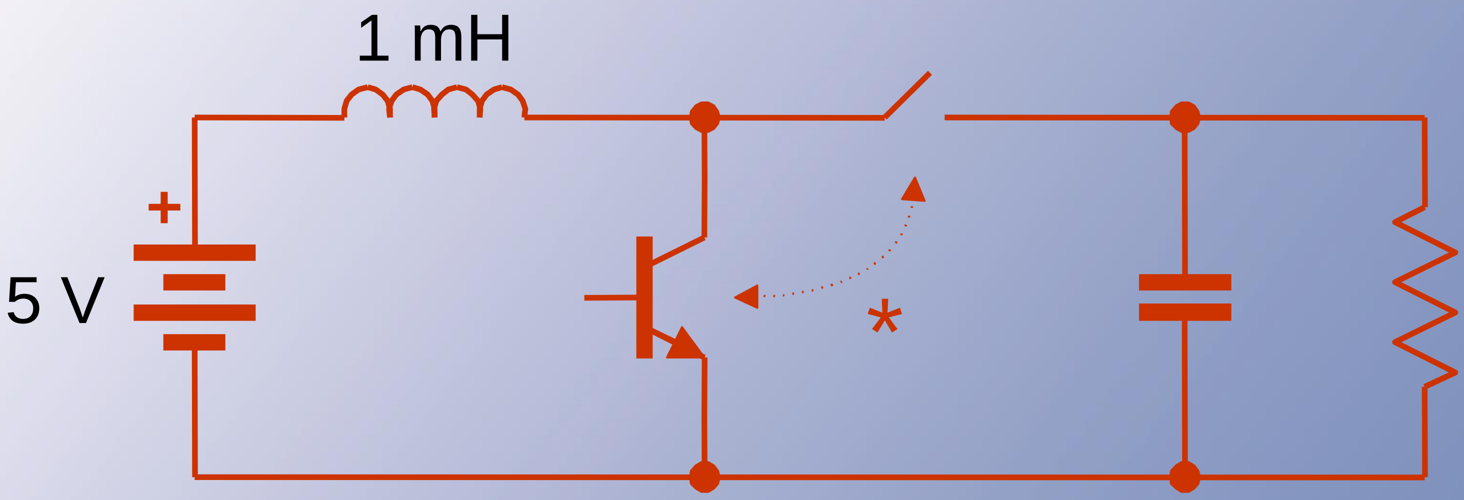
$$v_L = L\, di/dt$$

$$= (0.01\ \text{H})\ \frac{-10\ \text{A}}{1\mu\text{s}}$$

$$\approx -100{,}000\ \text{V}$$

Reality of Circuit Laws

KCL: Must provide a current path for any current source.



* Switching must be coordinated correctly, so as not to remove the path for I_L.

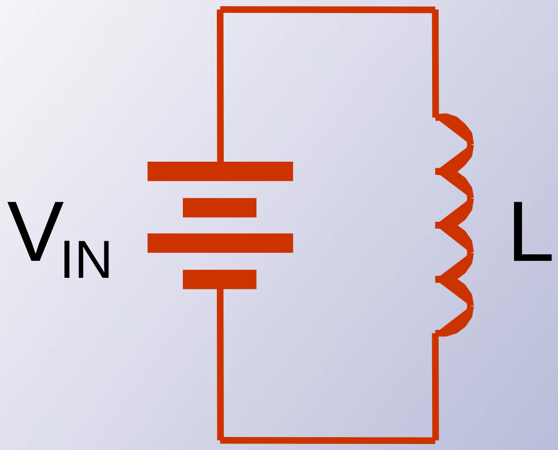
Reality of Circuit Laws

- Attempts to violate KCL can generate extreme voltages, as current tries to maintain its flow.
- It is hard to protect against this – fuses do not help.
- KVL “violations” are reasonable easy to avoid.
- KCL is more problematic in practice.

Implications for Storage

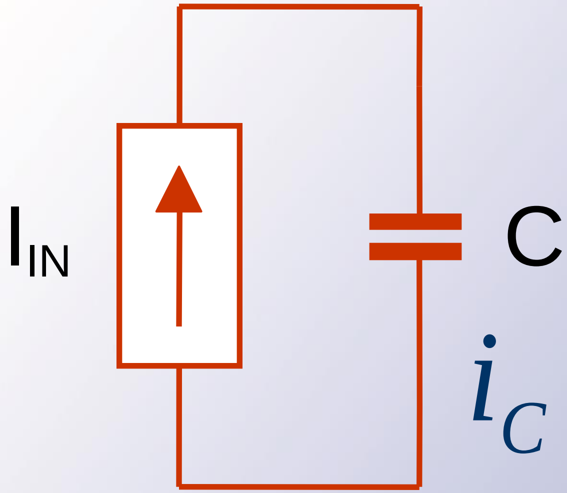
- If a fixed voltage is applied to an inductor, current rises without limit.
- This is like a short circuit, although it is OK for a short time.
- If a fixed current is applied to a capacitor, charge rises without limit.

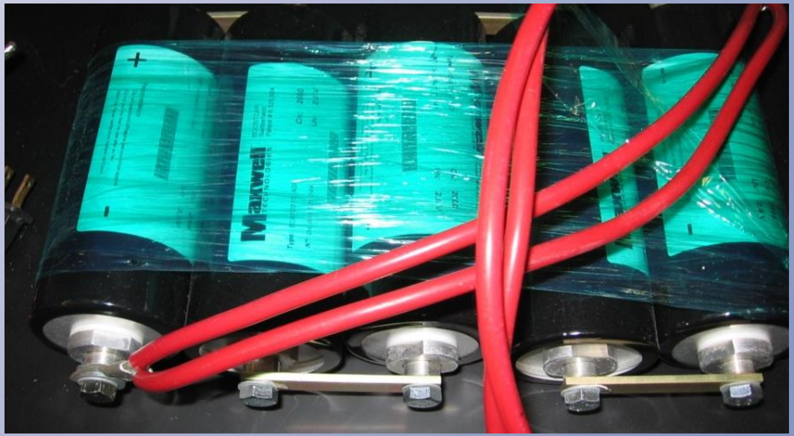
Implications for Storage



$$v_L = L\frac{di}{dt}$$

Okay in general, but must be time-limited, if we apply dc voltage to an inductor. In the ideal arrangement, there is no limitation on the current. *An inductor will not sustain dc voltage.*





$$i_C = C\frac{dv}{dt}$$

Okay, but must be time-limited.
Cannot apply dc current to a capacitor.

Capacitor will not sustain dc current.

Implications for Storage

- An inductor cannot sustain dc voltage over extended times.
- A capacitor cannot sustain dc current over extended times.

Implications for Storage

Since v_L must have no dc and i_C must have no dc, it must be true that:

$$\langle v_L \rangle = 0, \quad \langle i_C \rangle = 0.$$

These are key to circuit analysis: an inductor carries no average voltage; a capacitor carries no average current.

$$\langle v_L \rangle = 0$$

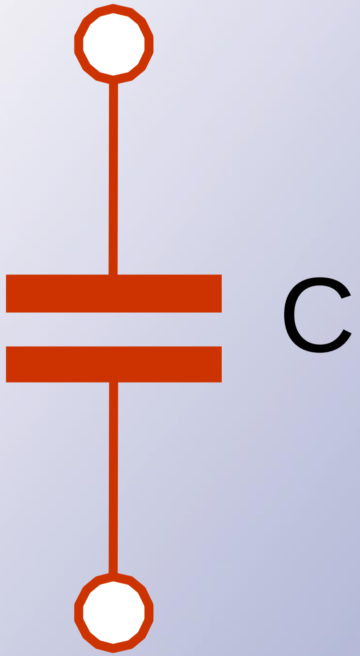
$$\langle i_C \rangle = 0$$

→ KVL and KCL

Implications for Switching

- We want ideal sources (source conversion concept).
- We cannot use a switch matrix for direct connection of voltage sources or of current sources.

Implications for Switching



$$i_C = C\frac{dv}{dt}$$

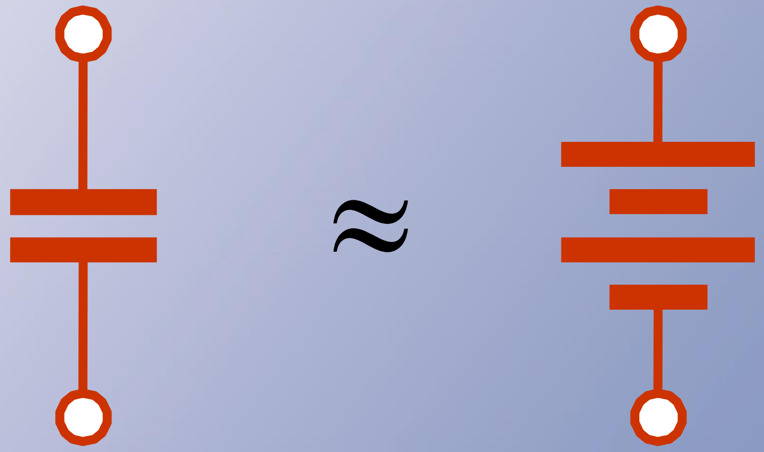
Over short times:

$$\frac{dv_C}{dt} = \frac{i_C}{C}$$

If C is large and i_C is bounded, then *dv/dt* can be as small as desired.

Implications for Switching

The previous statement means that a large capacitor acts as a voltage source over short times.



Implications for Switching

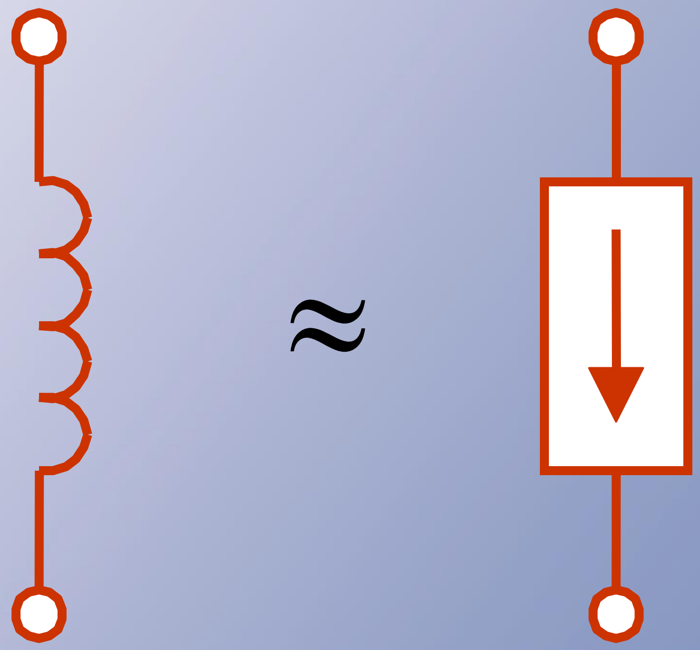
$$v_L = L\frac{di}{dt}$$

$$\frac{di_L}{dt} = \frac{v_L}{L}$$

If L is large and v_L is bounded, then *di/dt* can be made as small as desired.

Implications for Switching

The previous statement means that a large inductor acts as a current source over short times.



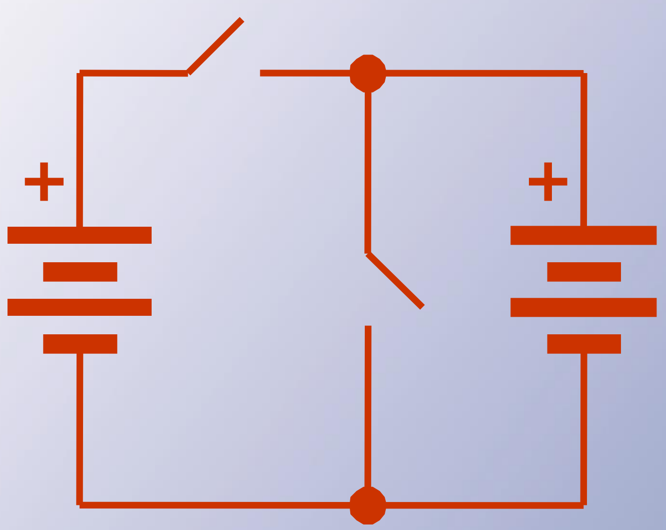
Source conversion

Implications for Switching

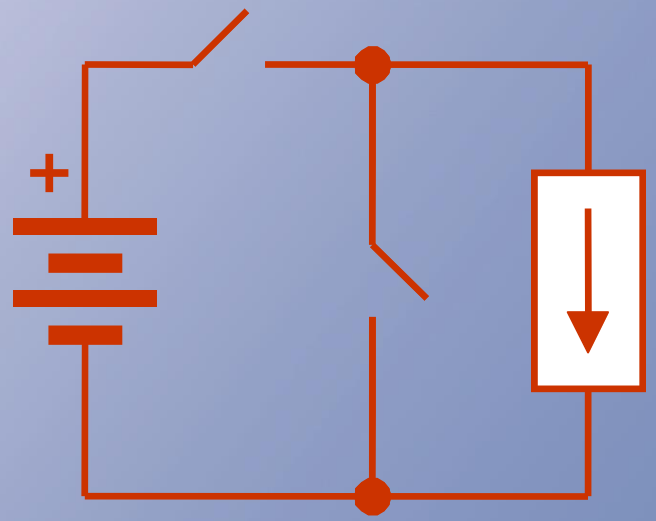
- Any useful converter must mix voltage and current sources.
- “Voltage converts to current,”
- “Current converts to voltage.”

Implications for Switching

The user wants ideal sources but we can't just use either V or I. Must mix these.

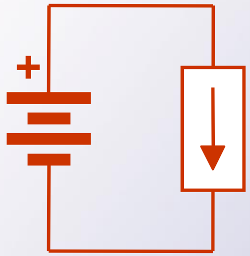


Problems

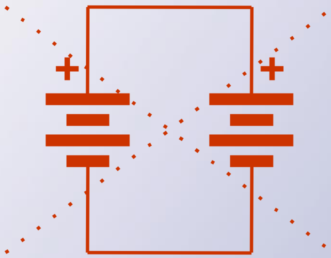


No problem

Implications for Switching

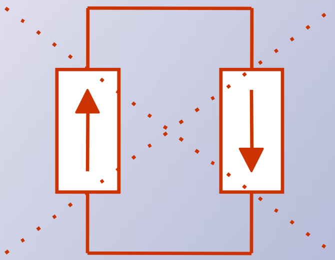


No problem



Problems

Example: Connecting two batteries in parallel, one ok, one discharged.



Problems

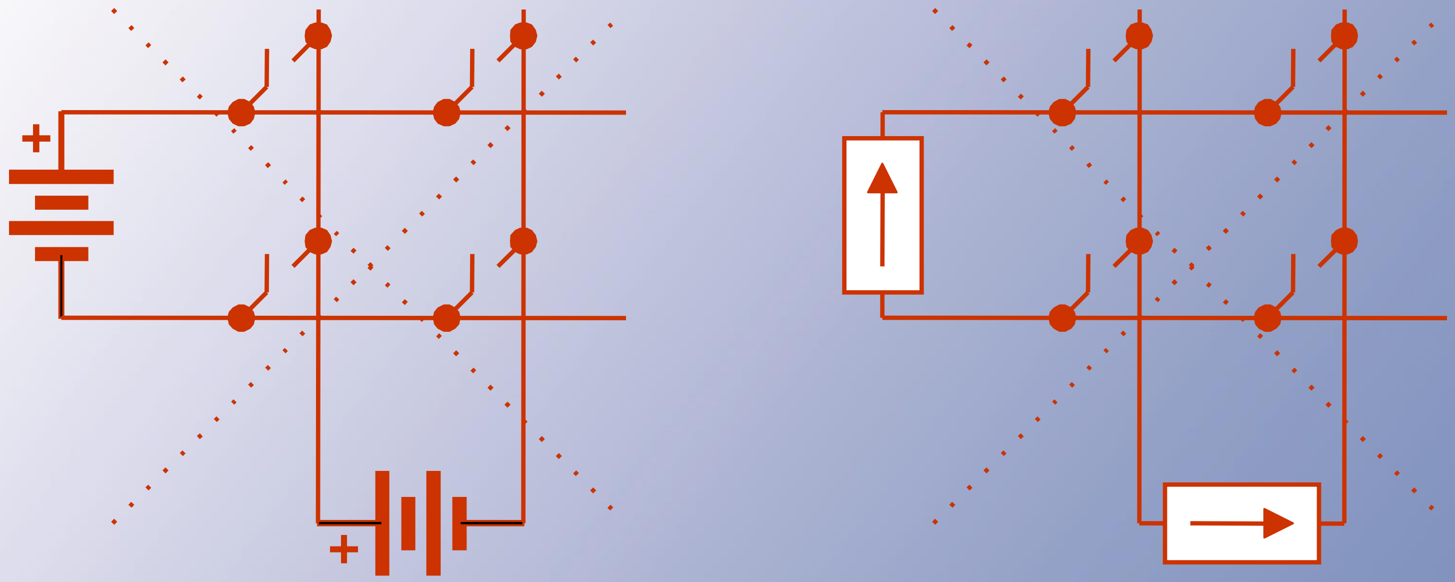
Implications for Switching

Source conversion

Voltage → Current

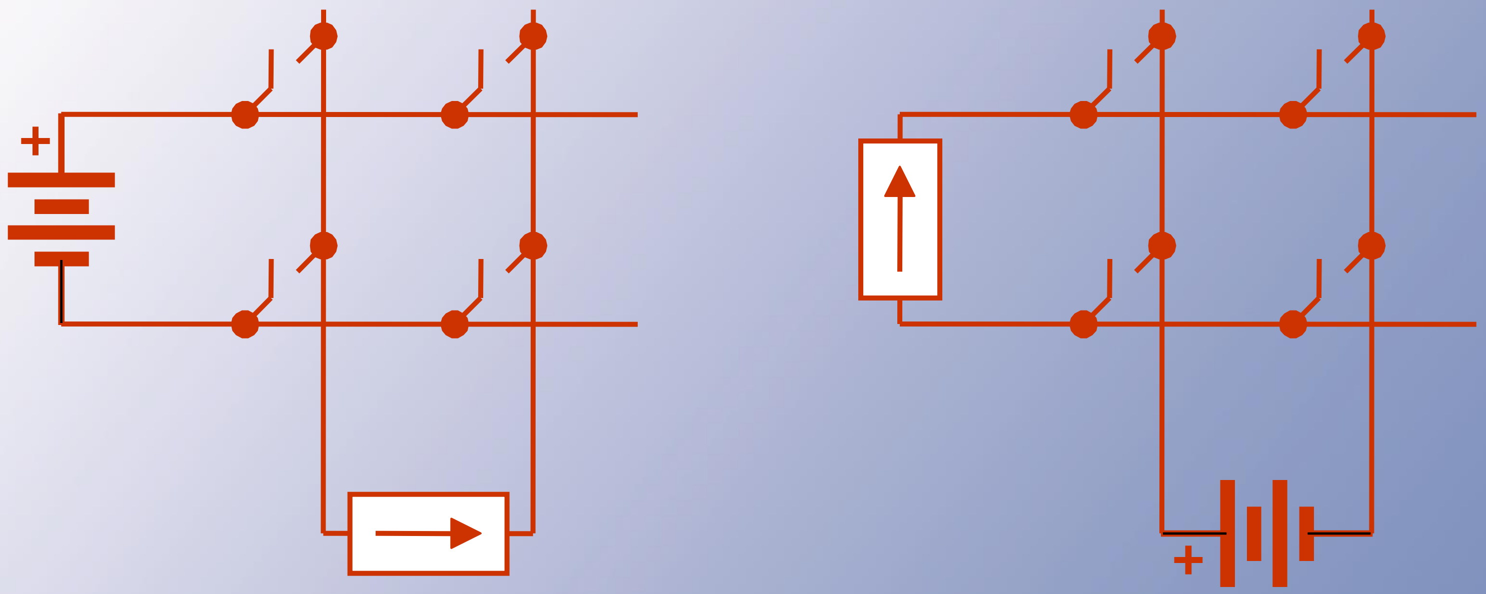
Current → Voltage

Implications for Switching



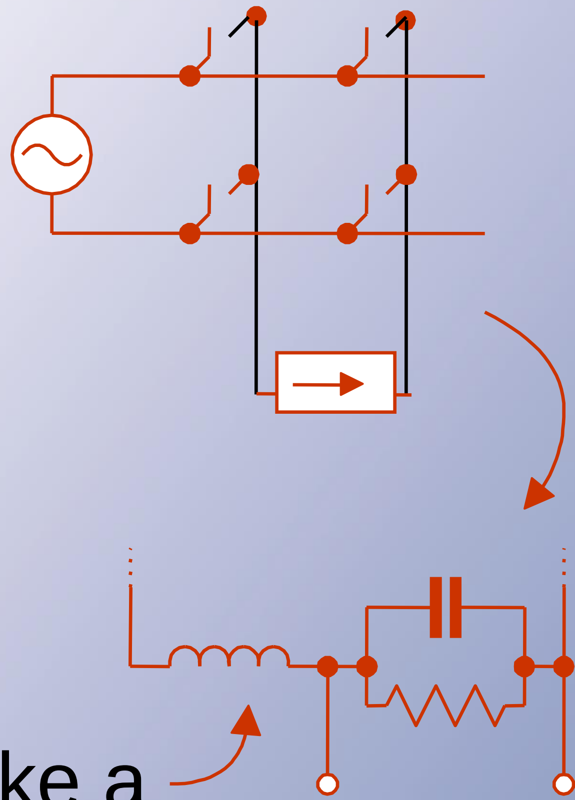
Because of KVL and KCL, neither of these can deliver useful energy.

Implications for Switching



These are valid combinations.

Implications for Switching
Possible useful configuration (a)



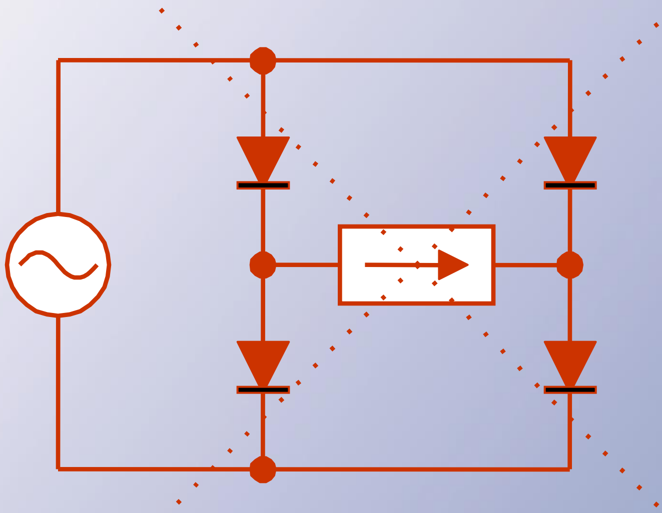
We can achieve (a) with something like (b).

Looks like a current source (b)

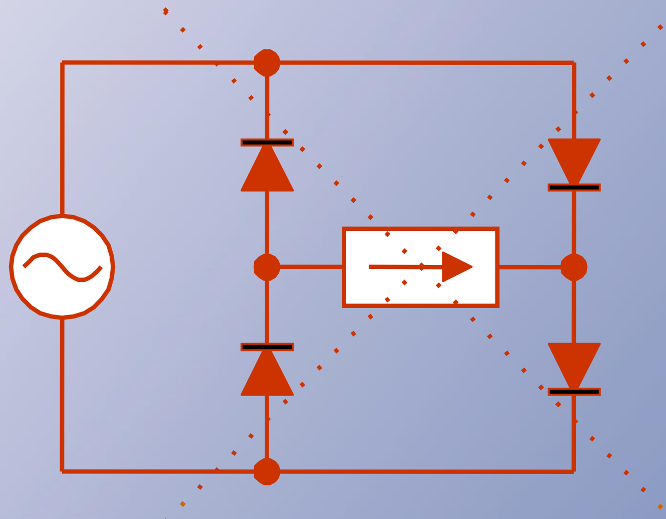
Diode Bridge Example

Of all possible connections, only one remains after KVL, KCL, and conversion requirements are met.

Diode Bridge Example

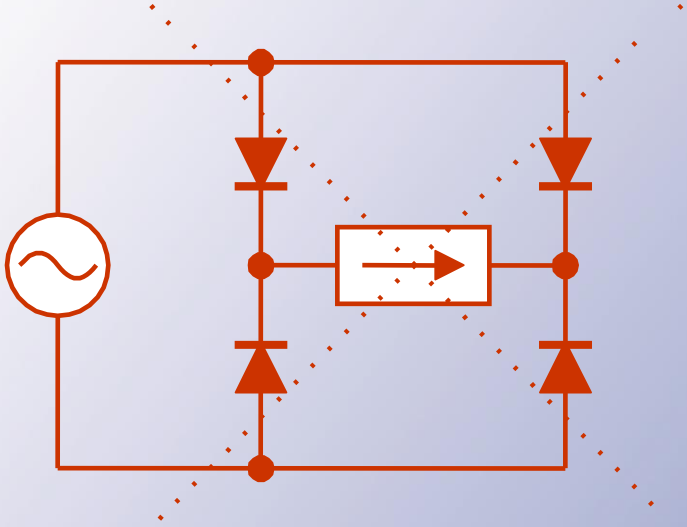


KVL problem

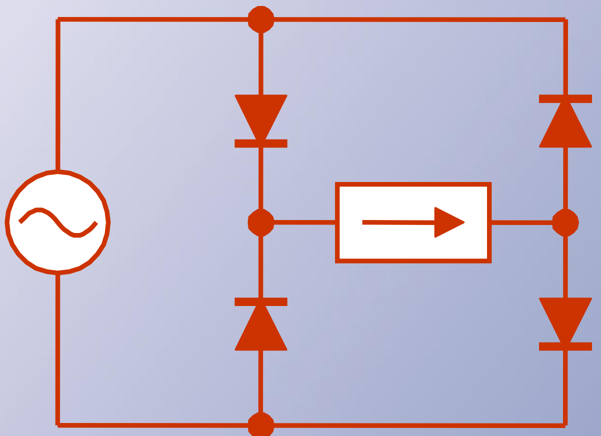


KVL problem

Diode Bridge Example



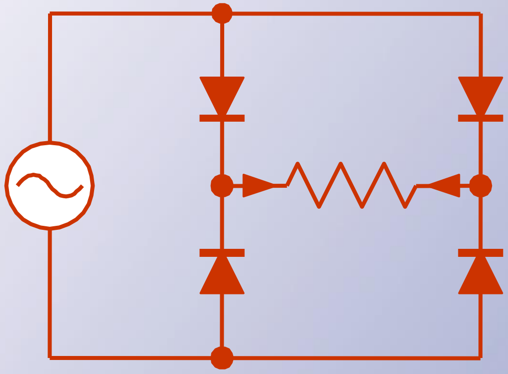
KCL problem



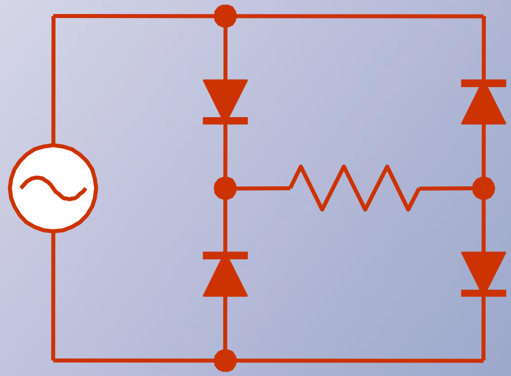
Diode Bridge.
The only combination
That does not violate
KCL or KVL

Diode Bridge Example

Can do the same with a resistor, rather than a current source. No KCL issues.



No KCL problem, but $I_{RESISTOR} = 0$ and therefore Power = 0



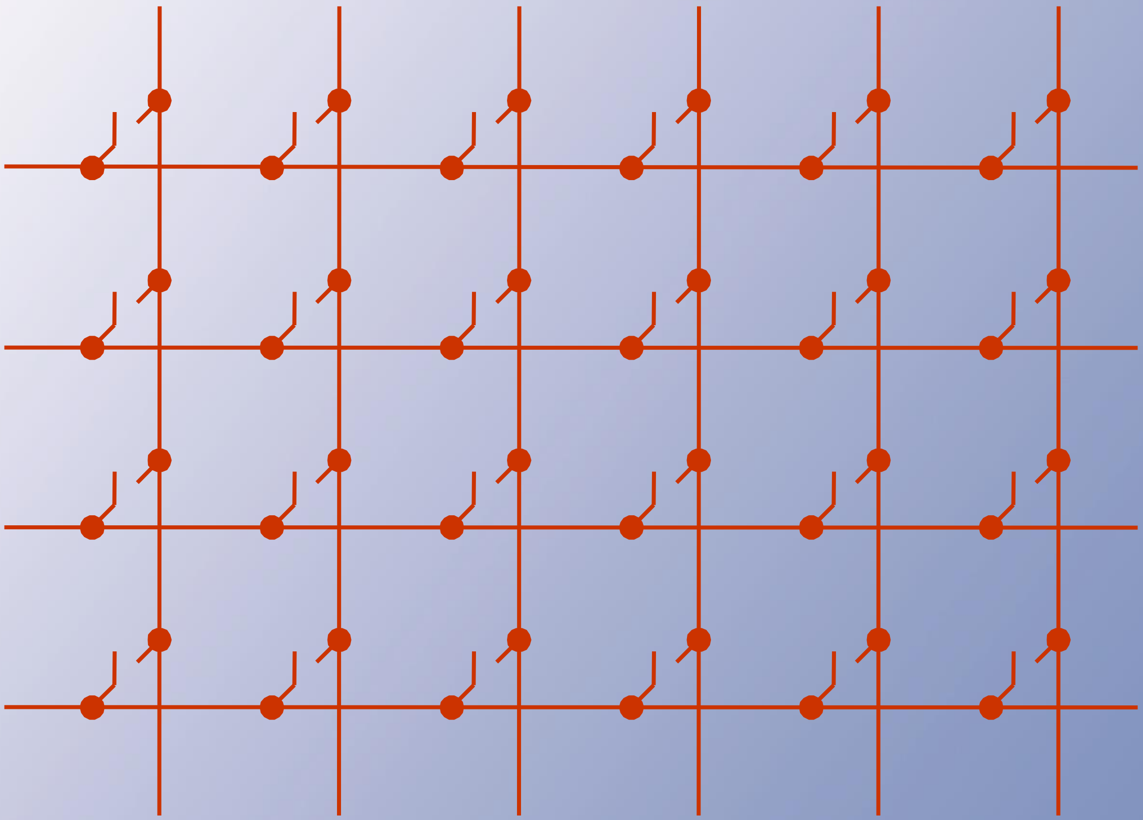
Diode Bridge. KVL and KCL are useful for us.

Summary of Analysis Rules

1. Conservation of energy.
2. Source conversion.
3. KVL: avoid voltage source interconnection.
4. KCL: provide current paths.
5. $\langle v_L \rangle = 0$
6. $\langle i_C \rangle = 0$

Switching Functions

m input lines

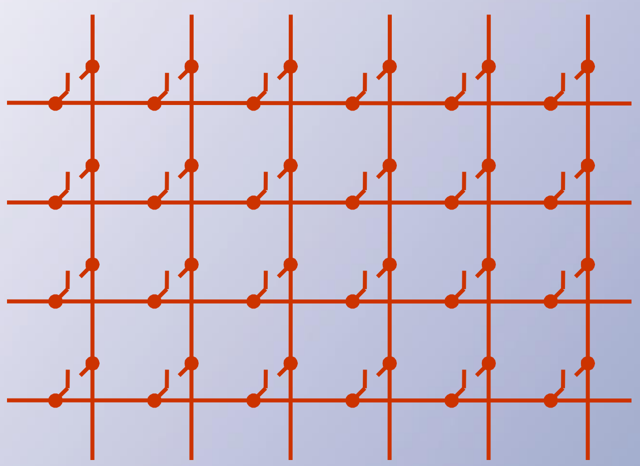


m x n switches

n output lines

Switching Functions

- The element at row *i* and column *j* represents switch *ij* and function $q_{ij}(t)$.
- The matrix Q has elements that correspond to each individual switch.



$$Q(t) = \begin{bmatrix} q & q & q & q & q \\ q & q & q & q & q \\ q & q & q & q & q \end{bmatrix}$$

$$q = 1 \text{ or } 0 \text{ at any time}$$

Switching Functions

- We have a physical switch matrix with *m* rows and *n* columns.
- Each switch is either on or off.
- Define a *switching function, q(t)* as 1 when a device is on, 0 when off.

Switching Functions

- Now, each physical switch is associated with a simple discrete function.
- Do not forget about time.
- We can define a *switch state matrix,* Q*(t).*

Switching Functions

- We can define our *software* problem in terms of choices of switching functions.
- We can find out the expected waveforms in many types of converters.

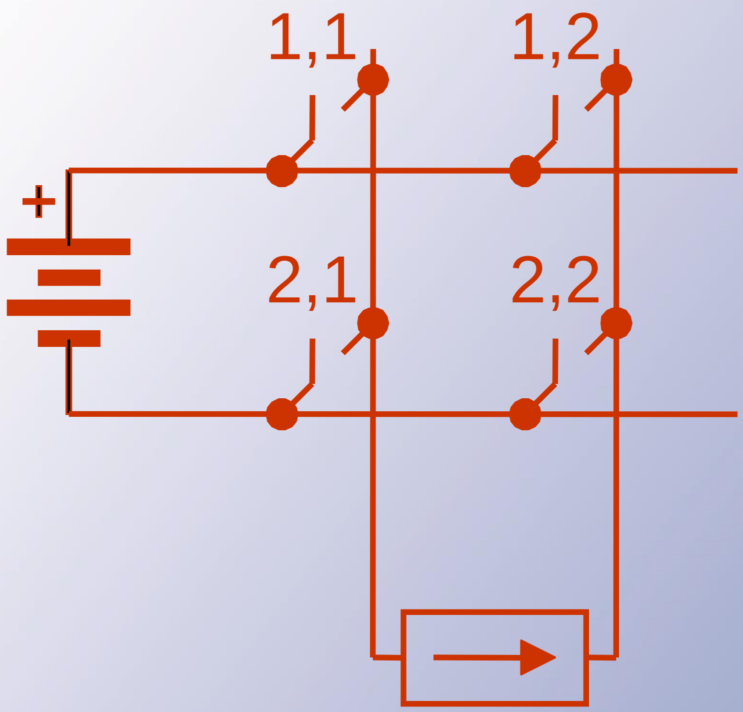
Summary So Far

- KVL and KCL represent restrictions on what we can do.
- Switching functions make our actions easier to quantify and analyze.

Switching Functions

- The functions support shorthand notation for KVL and KCL analysis.
- More important, they give mathematical expressions that represent converter action.

Example



"Generic" DC-DC converter.

q_{11}= 0 or 1

$$Q = \begin{bmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{bmatrix}$$

Example

KVL says we cannot have 1,1 and 2,1 ON or 1,2 and 2,2 ON, simultaneously.

KVL: Not 1,1 + 2,1 ON together.

Not 1,2 + 2,2 ON together.

KCL: Not 1,1 + 2,1 OFF together.

Not 1,2 + 2,2 OFF together.

Example

Compact description of the restrictions:

- KVL:

$$q_{11} + q_{21} \le 1$$
$$q_{12} + q_{22} \le 1$$

- KCL:

$$q_{11} + q_{21} \ge 1$$
$$q_{12} + q_{22} \ge 1$$

- Both:

$$q_{11} + q_{21} = 1$$
$$q_{12} + q_{22} = 1$$

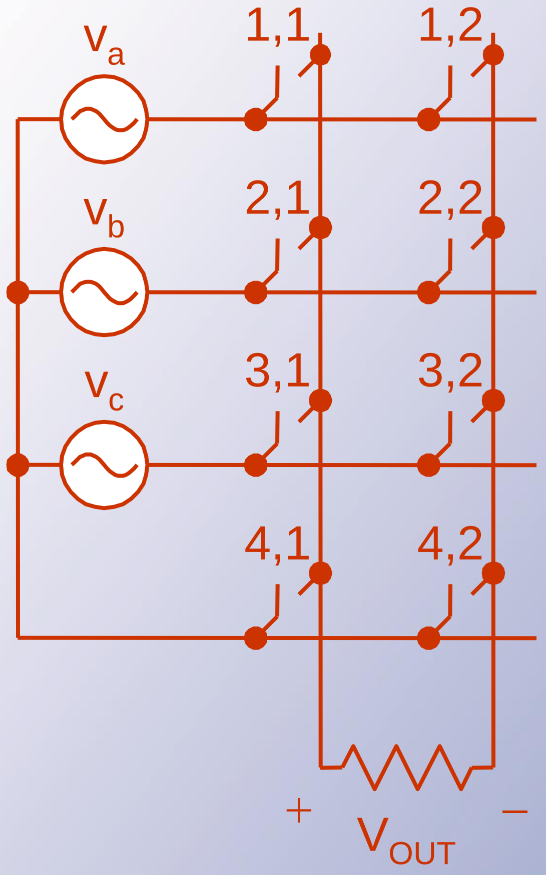
Example

The last two expressions can be written together as:

$$\sum_{i=1}^{N} q_{ij} = 1$$

"One and only one switch on at a time, for each column."

Example 2



Three AC voltage sources connected through a switch matrix to a load. The sources share a reference point.

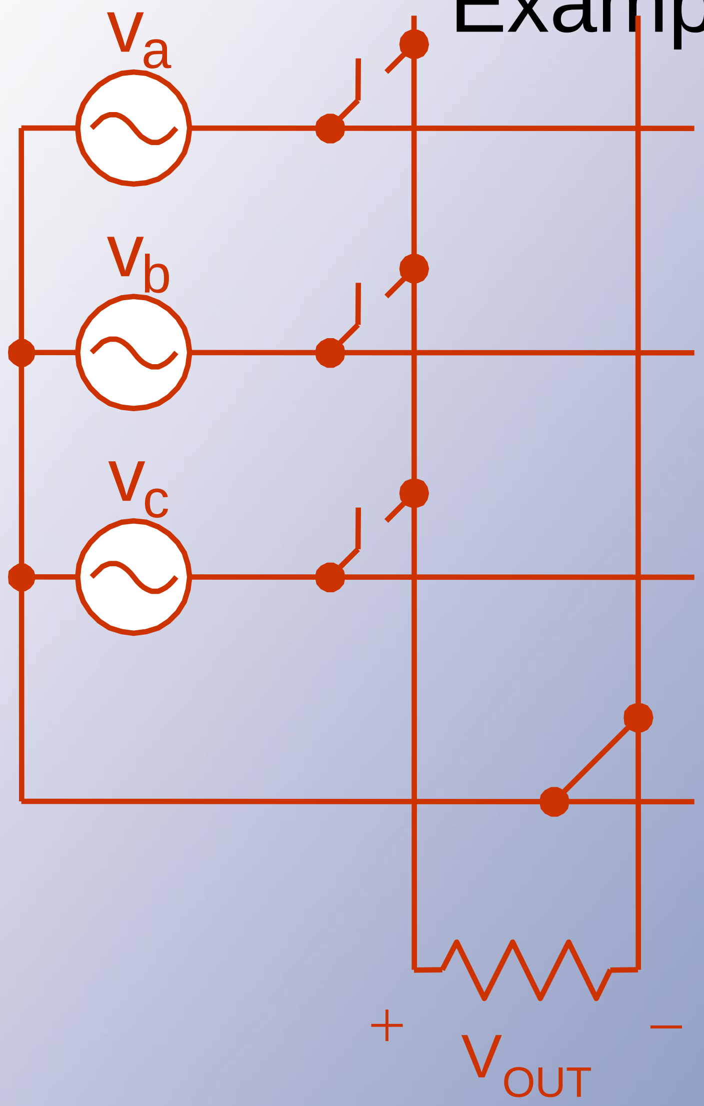
KVL : $\sum_{i=1}^{4} q_{i1} \leq 1 \qquad \sum_{i=1}^{4} q_{i2} \leq 1$

KCL : No restriction.

Switching Functions

- We can define our *software* problem in terms of choices of switching functions.
- We can find out the expected waveforms in many types of converters.

Example 3



$$q_{12} = 0$$

$$q_{22} = 0$$

$$q_{32} = 0$$

$$q_{41} = 0$$

$$q_{42} = 1$$

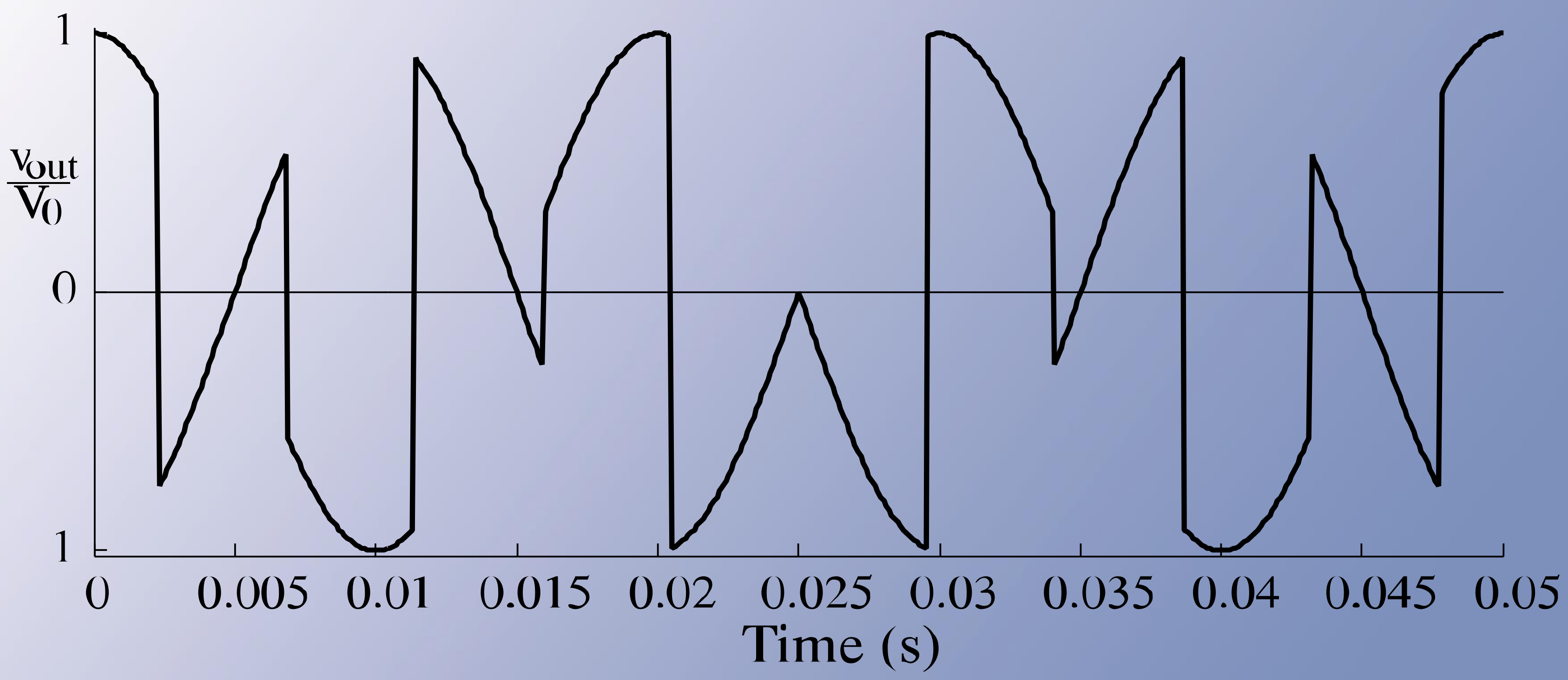
Example 3

$$v_{out} = q_{11}v_a + q_{21}v_b + q_{31}v_c + q_{41}0$$
$$- q_{12}v_a - q_{22}v_b - q_{32}v_c - q_{42}0$$

$$v_{out} = \sum_{i=1}^{4} q_{i1}v_i - \sum_{i=1}^{4} q_{i2}v_i$$

Example 3

The result is a "piecewise sinusoid."



So far

1) KVL + KCL:

Shorthand notation to understand the restrictions.

2) Outputs: (Voltage to current)

Given by products of switching functions and waveforms.

$$V_{out} = QV_{in}$$

$$I_{in} = QI_{out}$$

Switching Devices

- Characteristics of an **ideal** switch:
  - Any polarity of *v* or *i*, and no limits.
  - Can turn on or off at any time.
  - On, $v = 0$. Off, $i = 0$.
  - Acts instantly.

Switching Devices

- Real devices do not do any of this, of course.
- But even the best possible parts still have polarity limitations.

Switching Devices

- Some typical devices and capabilities are given below.
- For silicon PN devices, the typical forward drop is 1 V (*not* 0.7 V).

Diodes

- Current ratings exist from 1 A to nearly 10,000 A.
- Voltage ratings are from 10 V to 20kV.
- Not both at once (highest is about 5,000 A, 5,000 V).

Diodes

- Power junction devices with speeds of 20 ns to about 100 μs are used. As a general rule, large devices are slower.
- However, certain device grades are very slow.

Diodes

- Example: 1N4004 rectifier diode, 1 A, 400 V, speed is about 2 μs.
- MUR140 ultrafast rectifier diode, 1A, 400 V, speed is about 20 ns.

Diodes

- Schottky diodes are also widely used in power electronics.
  - **+ Lower forward drop**
  - **+ Very fast**
  - \- Lower voltage ratings
  - \- Higher leakage
- Emerging: SiC Schottkys, rated to 600 V or more (e.g. 600 V, 10 A, extremely fast)

Bipolar Transistors

- Rarely used as power switches now.
- IGBTs have replaced them in nearly all applications.

BJT

- Speed depends on absolute and relative rating.
- Typical devices (several amps and above) switch in 500 ns to a few tens of microseconds.

FET

- Most power devices are enhancement types that require a few volts between gate and source to turn on.
- Faster than BJTs, but lower ratings.
- Easy to use in parallel for high current.

FET

- Individual devices to about 100 A and 1000 V.
- Maximum power (single device) is about 10 kW.
- Modular packages with multiple devices can reach 500 A.

FET

- Power FETs are constructed as millions of small devices in parallel.
- The process inherently adds a “reverse parallel” diode internally.

SCRs

- The generic term is *thyristor.*
- These are PNPN multi-junction devices.
- "Latching" behavior: either off, or on, with a gate pulse.
- The SCR acts like a diode when on.

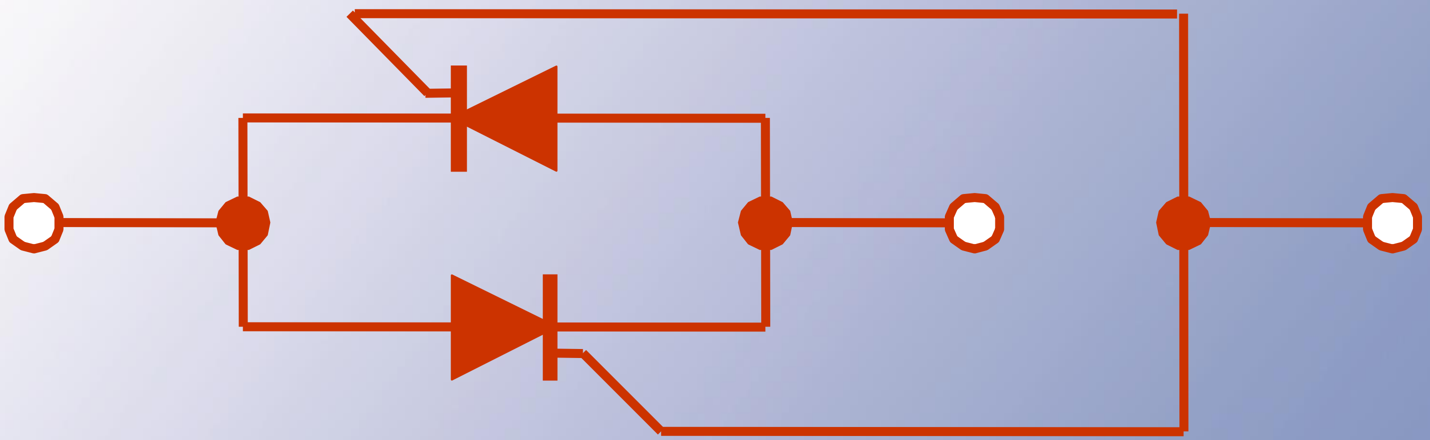
SCRs

- Ratings similar to diodes. Devices that can handle 6000 A and 6000 V simultaneously are available.
- Constructed as single-wafer devices.
- Relatively slow, 1 μs at best.

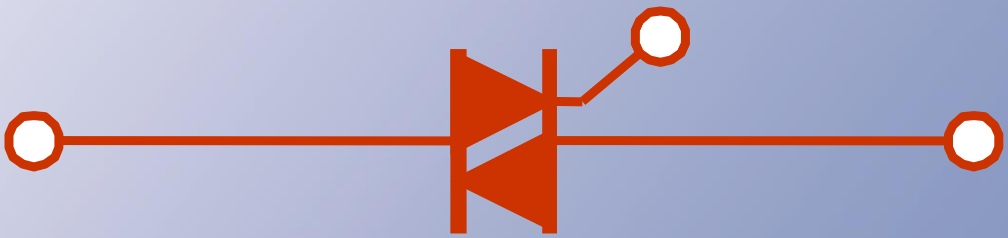
Other Thyristors

- GTO: an SCR that can be forced off with a negative gate pulse
- Light-fired SCR: an SCR that can be triggered with photons (from a laser)
- TRIAC: two SCRs in reverse parallel

Other Thyristors



TRIAC

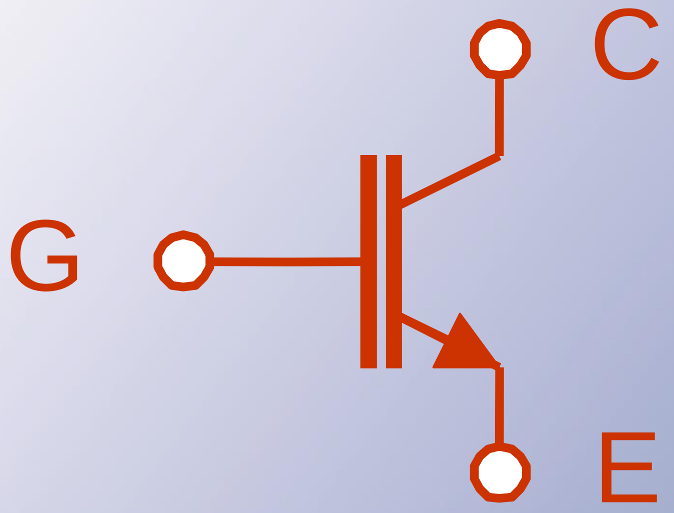


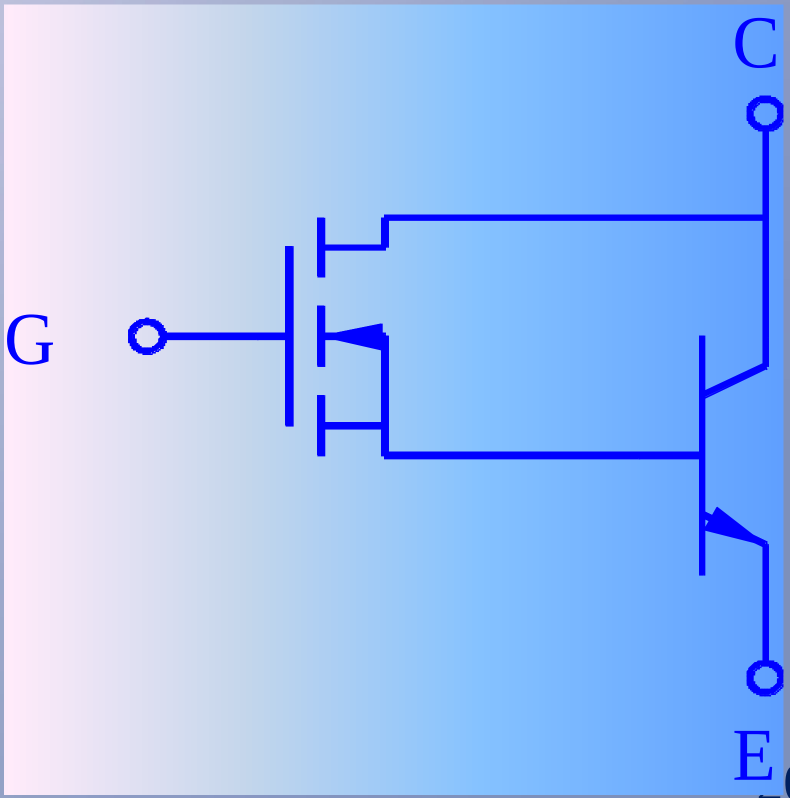
Combined Devices

- Combination devices are becoming popular.
- The oldest is the Darlington pair of BJTs.
- The IGBT is similar to a Darlington FET/BJT combination.

Combined Devices

- IGBT (Insulated-Gate Bipolar Transistor).
- Darlington combination of an FET and a BJT.



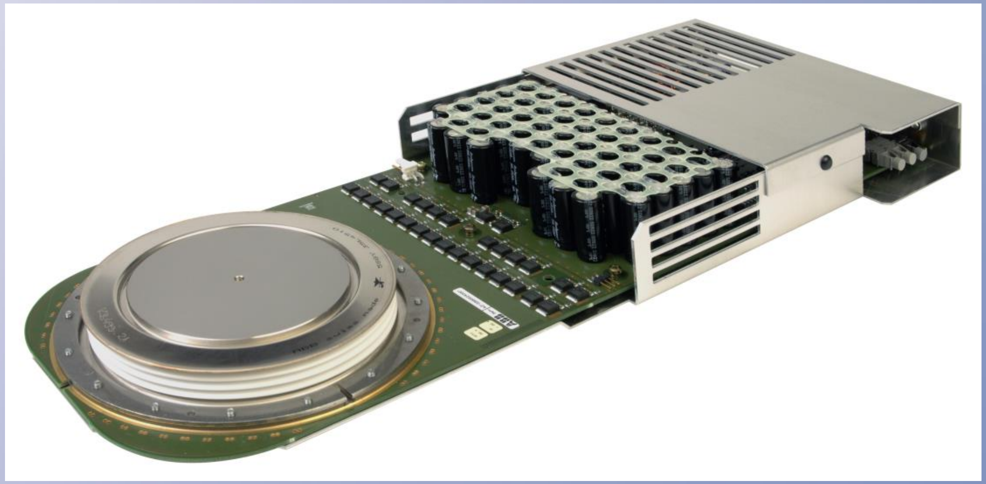


Combined Devices

- The IGBT combines gate behavior of FET with low voltage drop of BJT.
- Very popular for inverters.
- Devices are rated up to 1200 V and 1200 A.

Combined Devices

- IGBT is somewhat faster than BJT.
- High-power combinations are also available.
- Example: IGCT.



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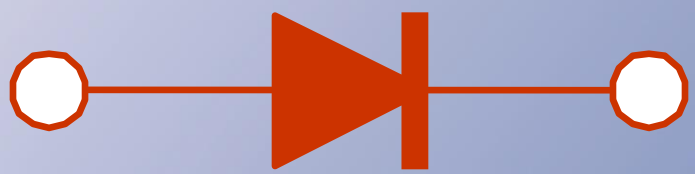
Restricted Switches

- Semiconductors (even the best ones) have polarity limitations.
- The *restricted switch* concept represents polarity effects in an ideal way.
- Classic example: Ideal diode conducts forward, blocks reverse.

Restricted Switch

- Idealized device with polarity

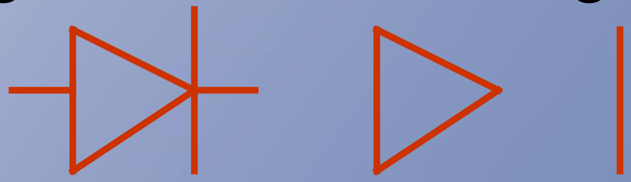
Ideal diode



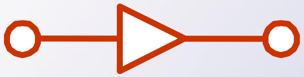
- $V_{forward} = 0 \quad I_{leak} = 0$

Restricted Switch Types

- Ideal diode
  - – No forward voltage drop
  - – No leakage current
  - – Action is determined by terminal conditions
  - – Symbol: triangle and bar
  - – *Forward conducting, reverse blocking switch* (FCRB)

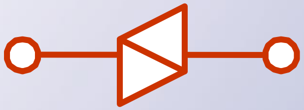


Restricted Switch Types

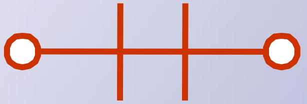


Triangle shows carrying direction

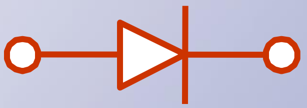
Bar shows blocking action



Conducts in both directions.
Does not block. Piece of wire



Prevents flow in both directions.
Open circuit

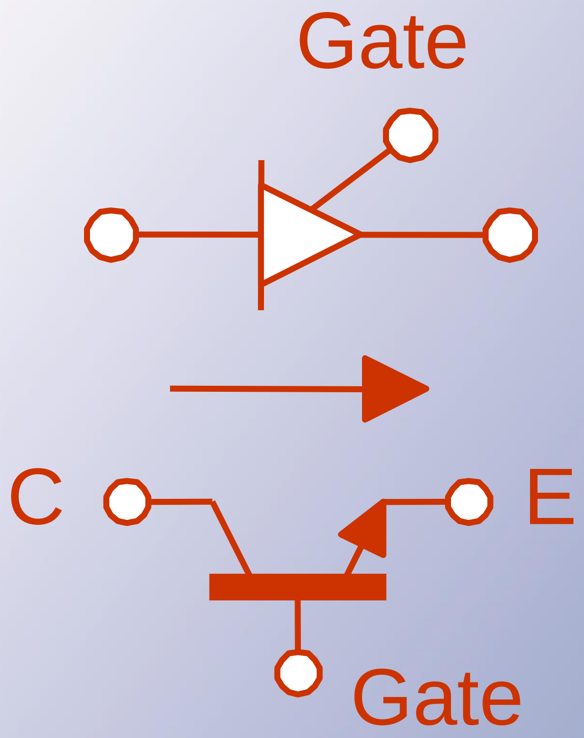


Example: Rectifier diode

FCFB

- Forward-conducting forward blocking (FCFB) switch
- Conducts or blocks in forward direction
- Needs a gate to establish operation
- Action is not allowed in reverse
- Describes a BJT or IGBT

FCFB

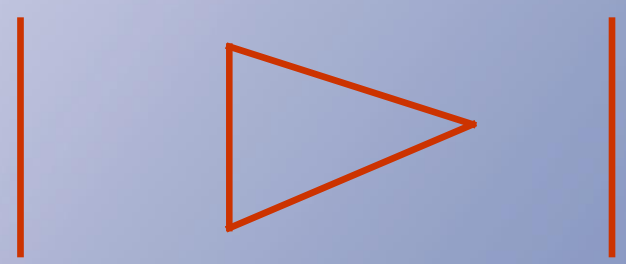


Symbol. Action not defined in reverse.

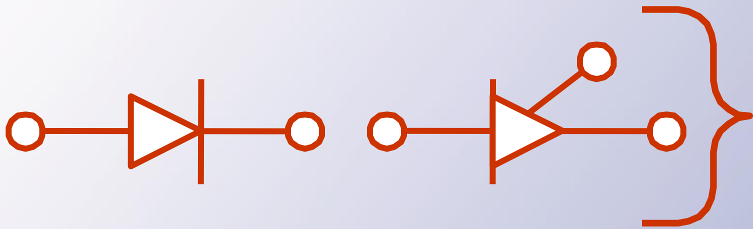
Example of implementation

FCBB

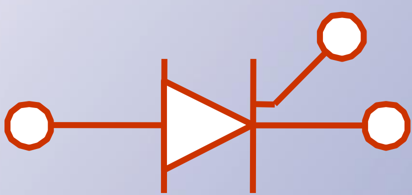
- Forward conducting, bidirectional blocking (FCBB)
- Always blocks in reverse, can carry or block forward.
- This describes a GTO, or a reverse-blocking IGBT.



FCBB

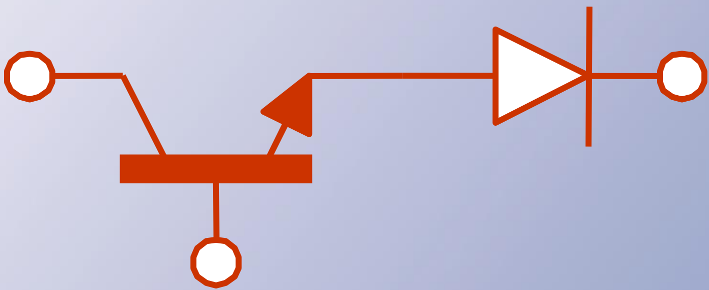


Unipolar switching devices



FCBB – Ex: GTO

Similar to SCR, but without gate restrictions

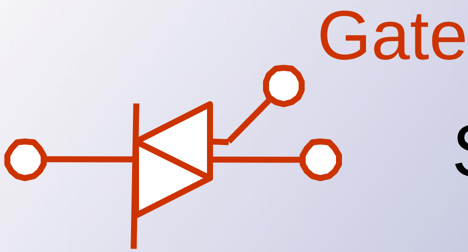


Possible implementation of FCBB

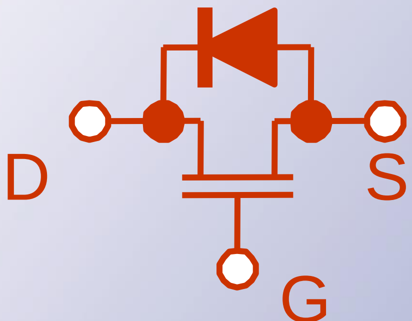
BCFB

- Bidirectional conducting, forward blocking (BCFB)
- Always allows reverse flow, but can carry or block forward
- Describes an ideal power FET

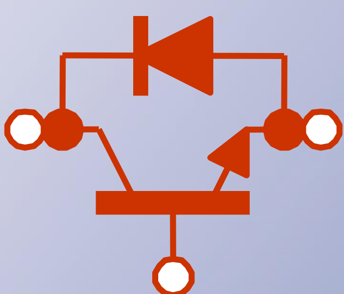
BCFB



Symbol



Possible implementation:
Power MOSFET

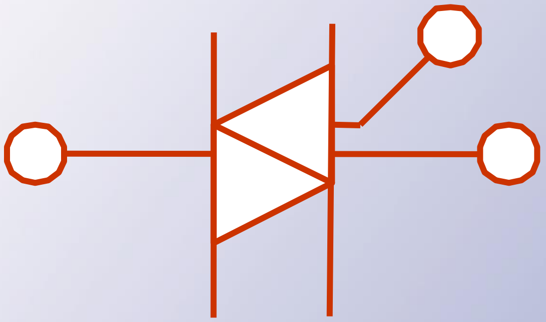


Possible implementation

BCBB

- Bidirectional conducting, bidirectional blocking (BCBB)
- Describes an ideal, or bilateral switch Sometimes called a bilat

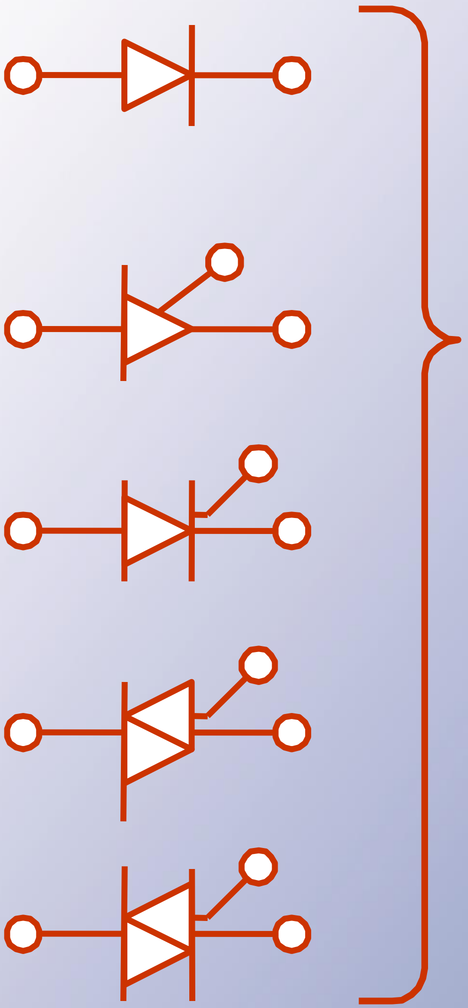
BCBB



BCBB

Ideal switch.

Restricted Switch Types

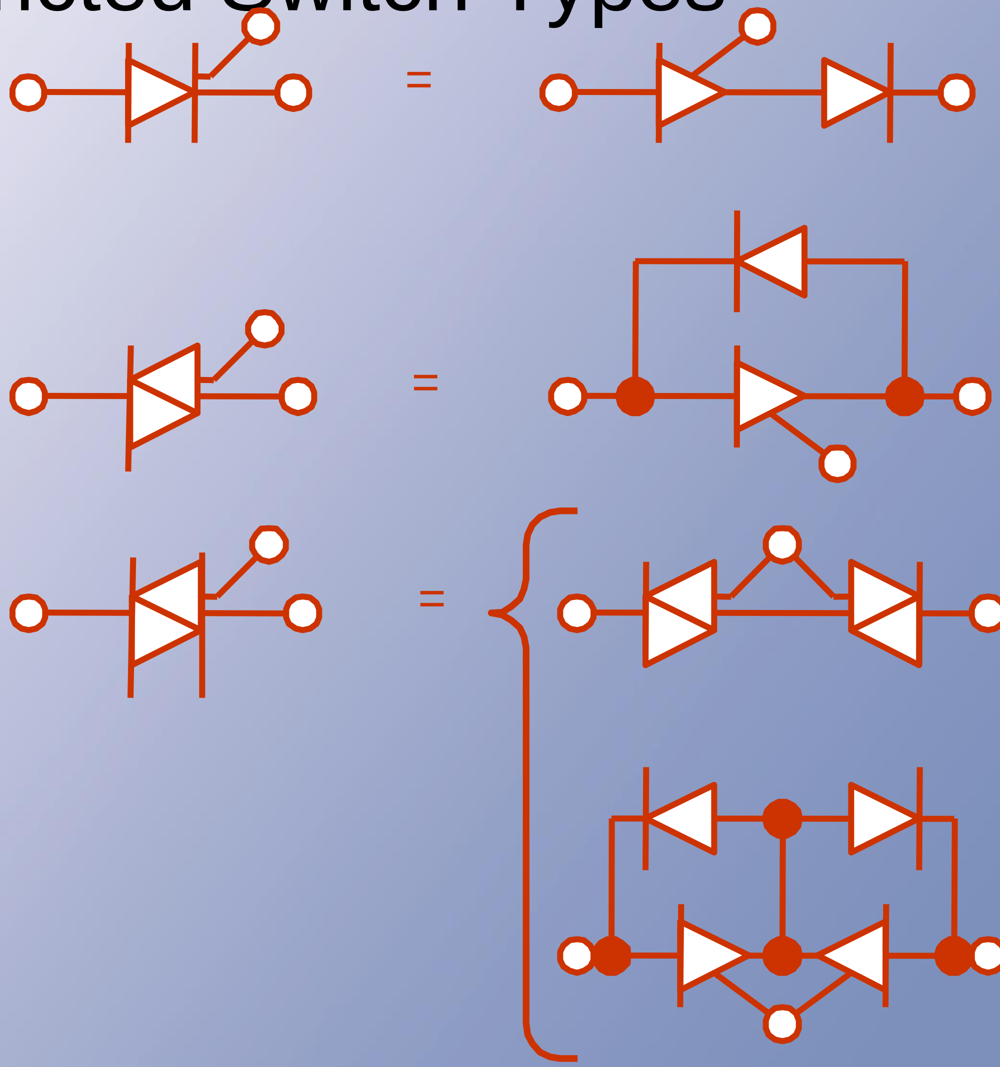


FCRB
(RCFB is flipped version)

Five restricted switches

Restricted Switch Types

Making bilateral switches



Switch Requirements

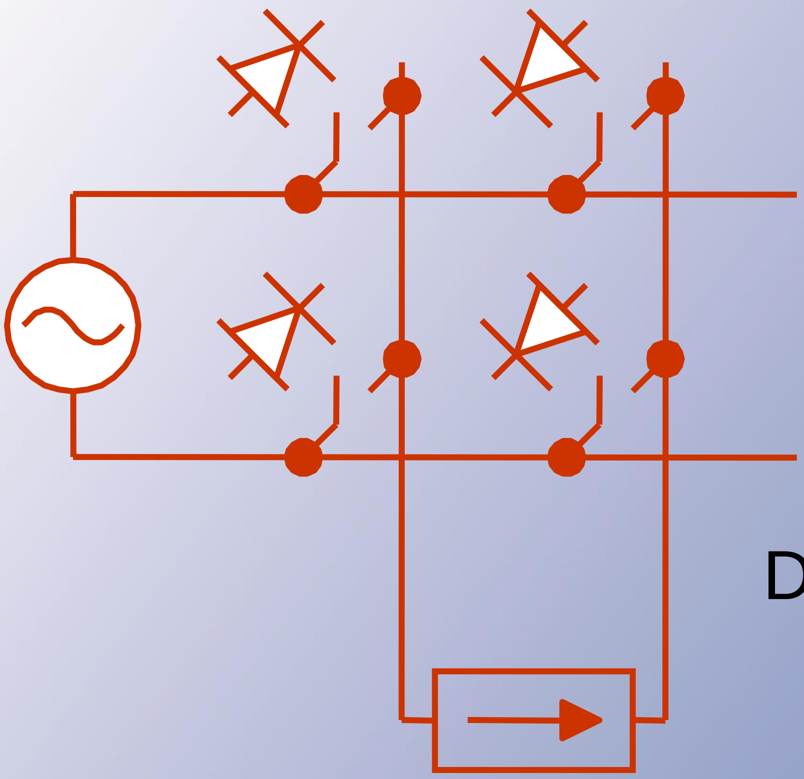
- The specific switch requirements can be identified through a direct process:
  - Check the current direction when the device is on.
  - Check the blocking polarity when the device is off.

Restricted Switch Action

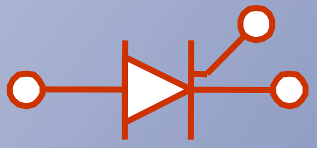
- Restricted switch types can be selected based on converter function.
- Example: Rectifier (for voltage to current conversion) should block ac voltage and carry dc current.

Restricted Switch Action

AC (V) to DC (I) converter



AC V: Need switches to block in both directions



DC I: Need switches to conduct in 1 direction.

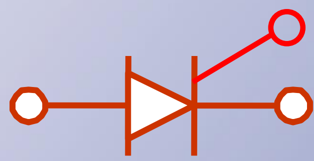
Restricted Switch Action

- Types of converters (voltage → current)
  - Dc-dc: Unilateral devices (FCFB, FCRB)
  - Ac-dc: FCBB
  - Dc-ac: BCFB
  - Ac-ac: BCBB

Restricted Switch Action

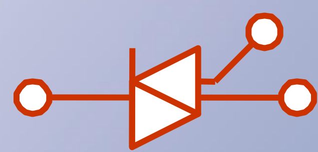
DC - DC Unipolar

AC (V) - DC (I)



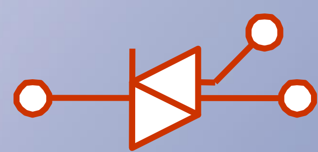
SCRs, GTOs

AC (I) - DC (V)



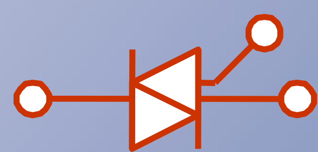
FET, BJT+Diodes, IGBT+Diodes

DC (V) - AC (I)



IGBT, FET

AC - AC



Bilateral

Restricted Switches

- The specific switch requirements can be identified through a direct process:
  - Check the current direction when the device is on.
  - Check the blocking polarity when the device is off.

Restricted Switches

- Choose a function (FCRB, FCFB, etc.) to match the need.
- Identify the function with a device (diode, FET, etc.).
- This is a basic approach for initial solution of the *hardware* problem.

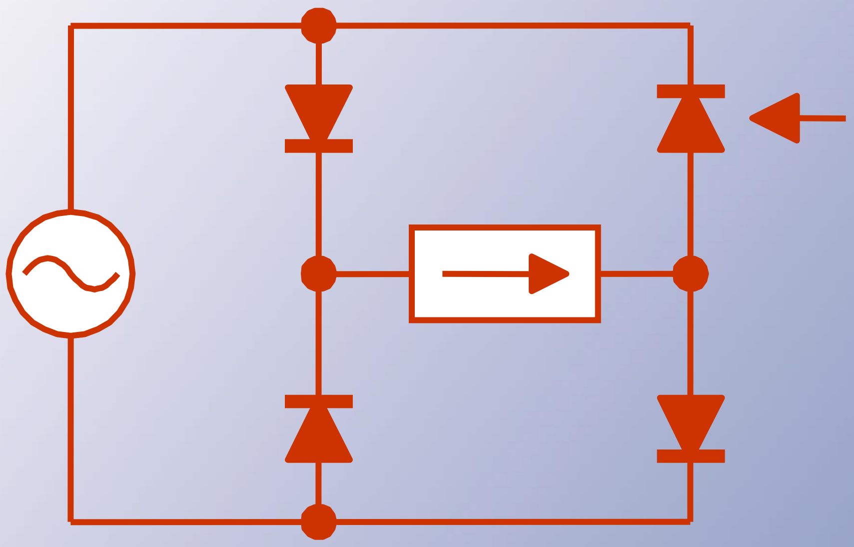
Restricted Switches

- Restricted switches are ideal except for polarity. No drop, no leakage, instant action, etc.
  - FCRB = ideal diode
  - FCFB = “ideal BJT”
  - BCFB = “ideal FET”

Analysis of Diode Circuits

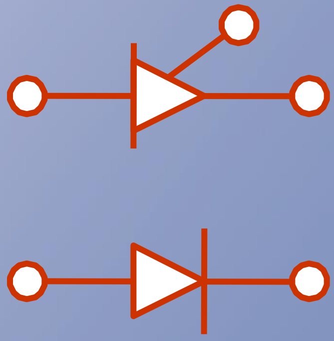
- The action of a diode (FCRB switch) is determined solely by the terminal conditions, not by an external gate.
- Once we know how to analyze diode circuits, the others follow.

Analysis of Diode Circuits



Depends on terminal relations

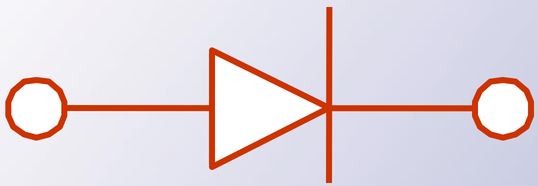
We can build any circuit with these two devices or combinations.



Analysis of Diode Circuits

- The reality is that we *do not know* how to perform a direct circuit analysis when ideal diodes are present.
- *But,* diodes can only be on or off.
- Need to perform a "piecewise" analysis.

Analysis of Diode Circuits



ON/OFF (which?)

Stays ON until i=0

Stays OFF until v=0

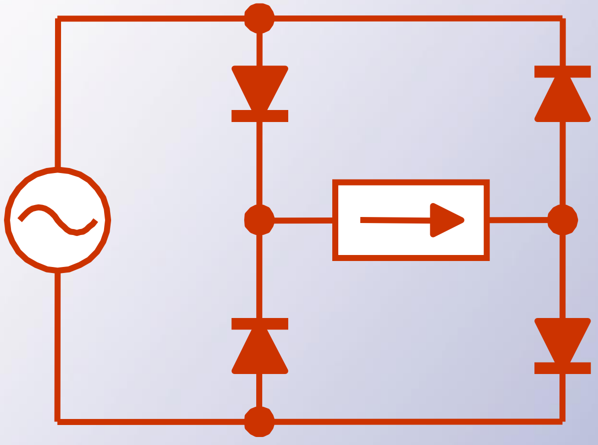
Analysis of Diode Circuits

- Diodes react only to terminal conditions:
  - If the device is on, it remains on while the current is positive.
  - If the device is off, it must turn on when the voltage is positive.

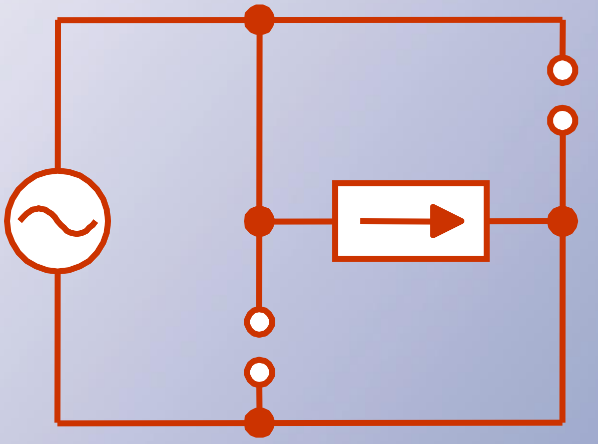
The Trial Method

- Any diode in a circuit must be either on or off.
- The diode state (on or off) determines the circuit configuration.
- Once the configuration is known, the circuit can be drawn and analyzed.

The Trial Method



Suppose we know which are on and which are off.



Configuration

The Trial Method

- Although the diode states are not arbitrary, *we are free to assign states* and then check the result.
- This is a trial and error method. But with a little practice, there are few errors.

The Trial Method

- Diodes satisfy KVL and KCL.
- Diodes always carry forward current, and block reverse voltage.
- In the trial method, we assign diode states, then check for consistency.

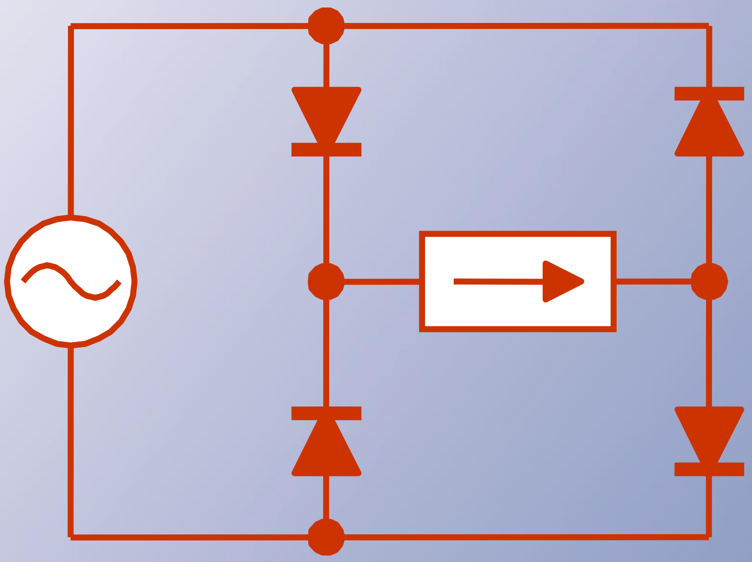
The Trial Method

- To check:
  - KVL and KCL must be satisfied.
  - On diodes must carry $i > 0$.
  - Off diodes must block $v < 0$.
- If the checks are OK, the assigned states are valid. If not, try another.

Keys

- Draw the configurations.
- Check polarities.
- If inconsistent, use polarities as a way to reassign states.

Basic Example
A simple rectifier connection



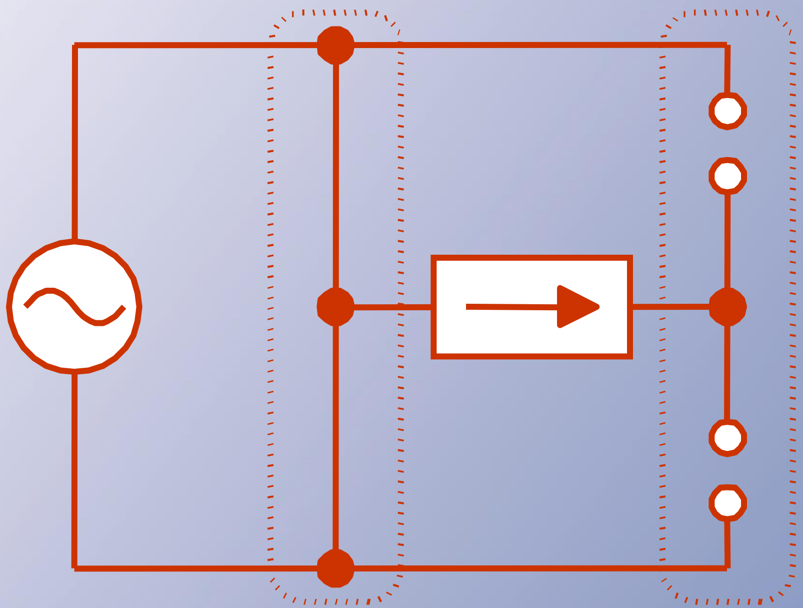
There are 16 configurations. Only one satisfies KVL, KCL, and power flow objectives.

Basic Example

- Try a few configurations with the trial method.

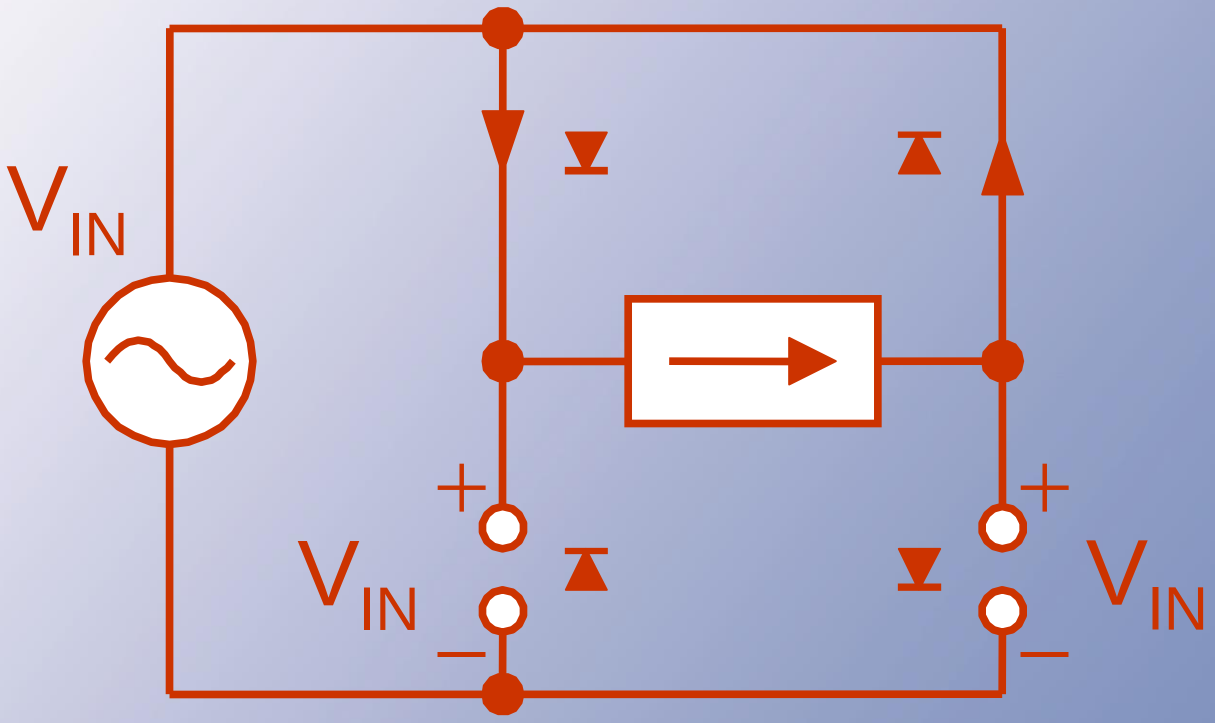
They cannot be on at the same time.

They cannot be off at the same time.



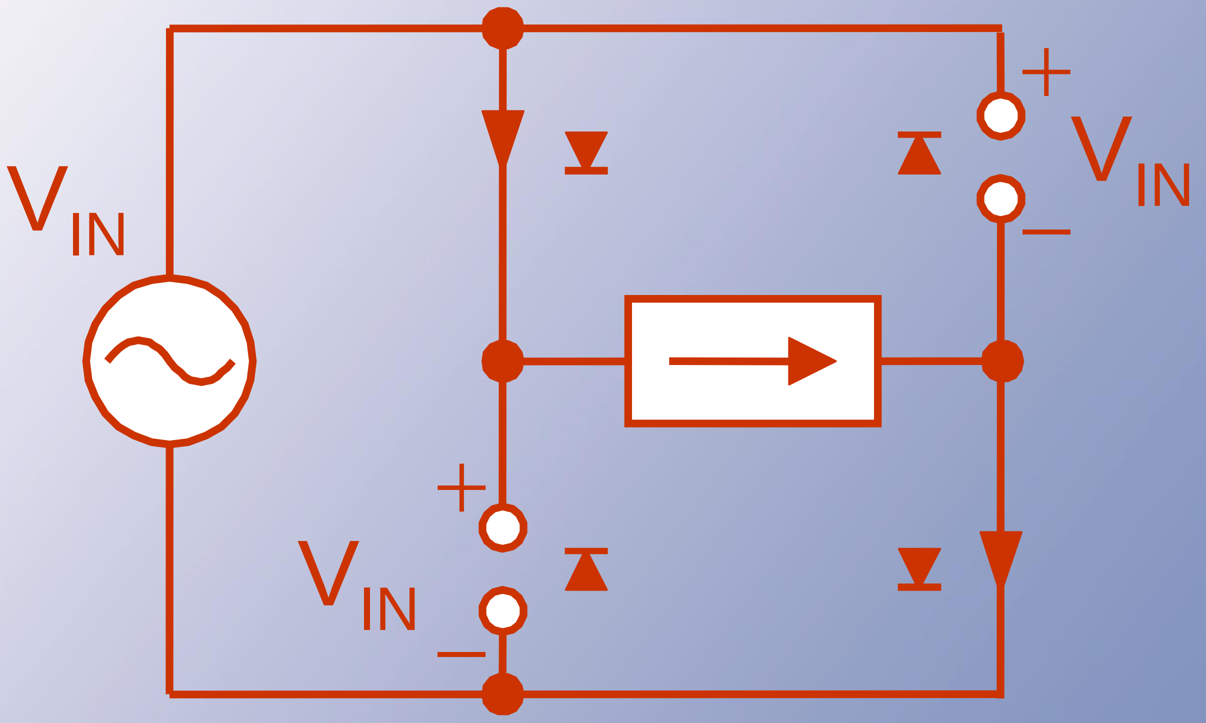
KVL and KCL not consistent

Basic Example



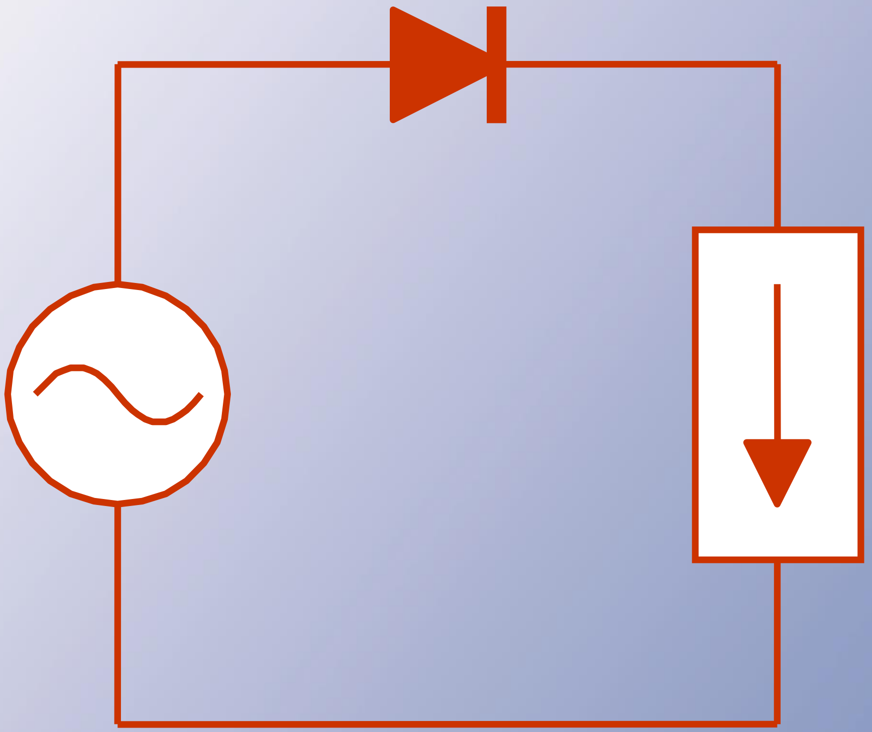
- KVL, KCL: Ok
- ON-State: Ok OFF-State: Inconsistent.

Basic Example



- KVL, KCL: Ok
- ON-State: Ok OFF-Voltage: Consistent if $V_{in}>0$

Basic Example



Diode is always on. Voltage is not relevant.
Current requires ON state.

The Trial Method

- The method works for complex circuits.
- Diodes are passive switches, since their action is governed by terminal conditions.
- Switches with gates are active.

The Trial Method

- Devices such as the FET can be analyzed as combinations:
  - If the gate is high, the device is on.
  - If the gate is low, the reverse parallel diode must be checked.
- The FCBB switch is similar (dual).

Summary

- Restricted switches, combined with the trial method, allow us to analyze idealized power converters.
- New power semiconductors try to approach ideal behavior.